Name: _____

PHYS2502 Mathematical Physics S23 Quiz #11 6 Apr 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

For the two matrices

$$\underline{\underline{L}}_x = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{and} \qquad \underline{\underline{L}}_y = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

(a) Determine if either, neither, or both of these matrices are Hermitian.

(b) Find the matrix given by $\underline{\underline{L}}_x \underline{\underline{L}}_y - \underline{\underline{L}}_y \underline{\underline{L}}_x$.

PHY2502 Mathematical Physics Quiz #11 Solution

For the two matrices

$$\underline{\underline{L}}_x = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \text{and} \qquad \underline{\underline{L}}_y = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

(a) Determine if either, neither, or both of these matrices are Hermitian.

(b) Find the matrix given by $\underline{\underline{L}}_x \underline{\underline{L}}_y - \underline{\underline{L}}_y \underline{\underline{L}}_x$.

Taking the transpose and complex conjugate you get the same matrix back for both $\underline{\underline{L}}_x$ and $\underline{\underline{L}}_y$, so both of these matrices are Hermitian.

$$\underline{\underline{L}}_x \underline{\underline{L}}_y = \frac{1}{2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & -i \end{bmatrix}$$

$$\underline{\underline{L}}_y \underline{\underline{L}}_x = \frac{1}{2} \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & i \end{bmatrix}$$

$$\text{so} \qquad \underline{\underline{L}}_x \underline{\underline{L}}_y - \underline{\underline{L}}_y \underline{\underline{L}}_x = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -i \end{bmatrix}$$