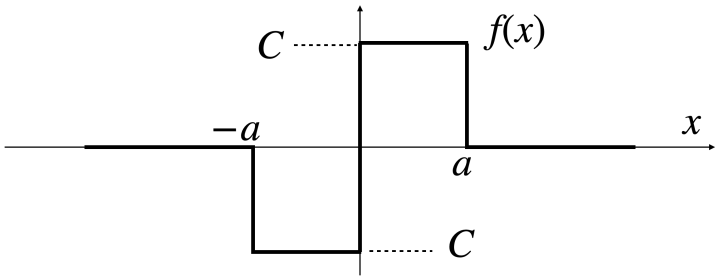
Name: _____

PHYS2502 Mathematical Physics S23 Quiz #10 30 Mar 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

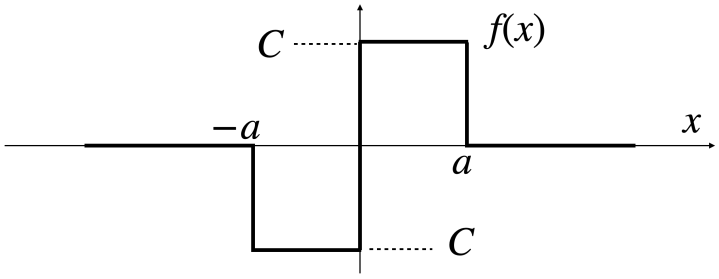
Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Find the Fourier Transform $A(k)$ for the pulse shape below, where C and a are constants:



PHY2502 Mathematical Physics Quiz #10 Solution

Find the Fourier Transform $A(k)$ for the pulse shape below, where C and a are constants:



$$\begin{aligned}
A(k) &= \int_{-\infty}^{\infty} e^{-ikx} f(x)\,dx = \int_{-\infty}^{\infty} [\cos(kx) - i\sin(kx)] f(x)\,dx \\
&= -i\int_{-a}^{a} \sin(kx) f(x)\,dx \\
&= -2i\int_{0}^{a} C\sin(kx)\,dx \\
&= 2iC\,\frac{1}{k}\cos(kx)\Big|_0^a \\
&= 2iC\frac{1}{k}\left[\cos(ka) - 1\right] \\
&= -4iC\frac{1}{k}\sin^2\left(\frac{ka}{2}\right)
\end{aligned}$$

Alternatively, we can just integrate the exponential:

$$\begin{aligned}
A(k) &= \int_{-\infty}^{\infty} e^{-ikx} f(x)\,dx = \int_{-a}^{0} (-C) e^{-ikx}\,dx + \int_{0}^{a} (C) e^{-ikx}\,dx \\
&= -C\,\frac{1}{-ik} e^{-ikx}\Big|_{-a}^{0} + C\,\frac{1}{-ik} e^{-ikx}\Big|_{0}^{a} \\
&= \frac{C}{ik}\left(1 - e^{ika}\right) - \frac{C}{ik}\left(e^{-ika} - 1\right) \\
&= \frac{C}{ik}\left(2 - e^{ika} - e^{-ika}\right) \\
&= 2iC\frac{1}{k}\left[\cos(ka) - 1\right] \\
&= -4iC\frac{1}{k}\sin^2\left(\frac{ka}{2}\right)
\end{aligned}$$