Name: ______________________________

PHYS2502 Mathematical Physics S23 Quiz #9 23 Mar 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

This quiz has **two parts**. Please answer both of them.

(a) An electric field in the x-direction permeates all space, and is of the form

$$\vec{E}(x,y,z) = E_0 \left(\frac{x}{L}\right)^2 \hat{\imath}$$

Find the amount of electric charge contained in a cube of side length L in the first octant, that is $x > 0$, $y > 0$, and $z > 0$, with a corner at the origin.

(b) Repeat this calculation, but now the electric field is

$$\vec{E}(x,y,z) = E_0 \left(\frac{x}{L}\right)^2 \hat{\jmath}$$

PHY2502 Mathematical Physics Quiz #9 <u>Solution</u>

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(a) Gauss' Law, with Gauss' Theorem, says that

$$Q_{\text{enc}} = k \oint_S \vec{E} \cdot d\vec{S} = k \int_V \vec{\nabla} \cdot \vec{E}\, dV$$

where $k = 1/4\pi$ in CGS units or $k = \epsilon_0$ in SI units. For the surface integral, realize that $\vec{E}$ only has a nonzero dot product along the faces parallel to the yz plane, and $x = 0$ on one of them, and $x = L$ on the other, so only one face contributes. The result is

$$\oint_S \vec{E} \cdot d\vec{S} = E_0 \left(\frac{L}{L}\right)^2 L^2 = E_0 L^2$$

If instead you want to use Gauss' theorem, we have

$$\int_V \vec{\nabla} \cdot \vec{E}\, dV = \int_0^L dx \int_0^L dy \int_0^L dz\, \frac{E_0}{L^2} 2x =, \frac{E_0}{L^2} \int_0^L 2x\, dx\, L\, L = E_0 L^2$$

which is, of course, the same as using the surface integral. Thereforre

$$Q_{\text{enc}} = \frac{1}{4\pi} E_0 L^2 \text{ (CGS)} \qquad \text{or} \qquad Q_{\text{enc}} = \epsilon_0 E_0 L^2 \text{ (SI)}$$

(b) The integrals are zero, so the enclosed charge is zero. For the surface integral, you integrate over the two faces parallel to the xz plane, but you get the same answer with opposite signs on each. For the volume integral, it is simple to see that $\vec{\nabla} \cdot \vec{E} = 0$.