Name: ______________________________

PHYS2502 Mathematical Physics S23 Quiz #8 16 Mar 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

Two different straight lines containing points $\vec{r} = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$ are given by the equations

$$\vec{r} = \vec{m}_1 t + \vec{r}_0 \qquad \text{and} \qquad \vec{r} = \vec{m}_2 t + \vec{r}_0$$

where $\vec{m}_1 = -2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$, $\vec{m}_2 = -2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, and $\vec{r}_0 = \hat{\imath} + \hat{\jmath} + \hat{k}$. Find the equation of the plane that contains both of these lines, in the form $Ax + By + Cz = D$.

PHY2502 Mathematical Physics Quiz #8 Solution

Two different straight lines containing points $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ are given by the equations

$$\vec{r} = \vec{m}_1 t + \vec{r}_0 \qquad \text{and} \qquad \vec{r} = \vec{m}_2 t + \vec{r}_0$$

where $\vec{m}_1 = -2\hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{m}_2 = -2\hat{i} - 3\hat{j} + 4\hat{k}$, and $\vec{r}_0 = \hat{i} + \hat{j} + \hat{k}$. Find the equation of the plane that contains both of these lines, in the form $Ax + By + Cz = D$.

The equation for a plane is $(\vec{r} - \vec{r}_0) \cdot \hat{n} = 0$ where $\vec{r}_0$ is a point in the plane and $\hat{n}$ is a normal (unit) vector. Both lines pass through $\vec{r}_0 = \hat{i} + \hat{j} + \hat{k}$, so we can use that as the point in the plane. The normal vector $\vec{n}$ has to be normal to both the direction of line #1 and line #2, so we can get it from the cross product of $\vec{m}_1$ and $\vec{m}_2$, namely

$$\begin{aligned} \vec{n} &= \vec{m}_1 \times \vec{m}_2 = (m_{1_y} m_{2_z} - m_{1_z} m_{2_y})\hat{i} + (m_{1_z} m_{2_x} - m_{1_x} m_{2_z})\hat{j} + (m_{1_x} m_{2_y} - m_{1_y} m_{2_x})\hat{k} \\ &= (12 - 6)\hat{i} + (4 + 8)\hat{j} + (6 + 6)\hat{k} = 6(\hat{i} + 2\hat{j} + 2\hat{k}) \end{aligned}$$

It's not necessary to normalize the normal vector, since you can just multiply through in the equation of plane by some constant, but just to be complete

$$|\vec{n}| = 6\sqrt{1 + 4 + 4} = 18 \qquad \text{so} \qquad \hat{n} = \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

To get to the standard form of a plane, we write $\vec{r} \cdot \hat{n} = \vec{r}_0 \cdot \hat{n}$, so

$$\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z = \frac{1}{3} + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$$

It's a little neater to multiply through by 3, that is

$$x + 2y + 2z = 5$$