Name: ______________________________

PHYS2502 Mathematical Physics S23 Quiz #4 9 Feb 2023

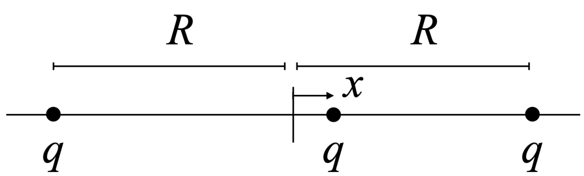
You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

You recall from some physics course that the force between two electric charges q_1 and q_2 is

$$F = k\frac{q_1 q_2}{d^2}$$

where k is a constant and d is the distance between the two charges. The force is repulsive if the signs of q_1 and q_2 are the same. Now consider the case of three identical charges q constrained to be on a line as shown here:



The outer two charges are fixed to the line, and are separated by a distance $2R$. The third charge can move along the line, where x measures its distance from the midpoint. Find the force on the middle charge in the limit $x \ll R$ and use this to find the (angular) frequency "ω" for small oscillations about the midpoint if the middle charge has mass m.

PHY2502 Mathematical Physics Quiz #4 <u>Solution</u>

You recall from some physics course that the force between two electric charges q_1 and q_2 is

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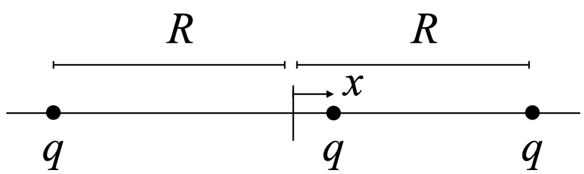

The outer two charges are fixed to the line, and are separated by a distance $2R$. The third charge can move along the line, where x measures its distance from the midpoint. Find the force on the middle charge in the limit $x \ll R$ and use this to find the (angular) frequency "ω" for small oscillations about the midpoint if the middle charge has mass m.

The force on the middle charge is positive from the charge on the left, and negative from the charge on the right, so

$$\begin{aligned} F = k\frac{q^2}{(R+x)^2} - k\frac{q^2}{(R-x)^2} &= k\frac{q^2}{R^2}\left[\frac{1}{(1+x/R)^2} - \frac{1}{(1-x/R)^2}\right] \\ &\approx k\frac{q^2}{R^2}\left[1 - 2\frac{x}{R} - \left(1 + 2\frac{x}{R}\right)\right] = -4k\frac{q^2}{R^3}x \end{aligned}$$

This has the form $F = -kx$ where $k \to 4kq^2/R^3$ is the stiffness of the spring, so

$$\omega^2 = \frac{\text{“}k\text{''}}{m} = k\frac{q^2}{mR^3} \qquad \text{and} \qquad \omega = \sqrt{k\frac{q^2}{mR^3}}$$

We can easily check the dimensionality of this result. The factor kq^2/R^2 has dimensions of force, namely MLT^{-2}. Dividing by mR leaves us with T^{-2}, and taking the square root indeed gives T^{-1} which is frequency.