Name: _____

PHYS2502 Mathematical Physics S23 Quiz #2 26 Jan 2023

You have fifteen minutes to complete this quiz. You may use books, notes, or computers you have with you, but you may not communicate with anyone other than the instructor.

Write your solution on this page, plus the back if necessary, and additional sheets if absolutely necessary. You must show the steps of your solution.

A pendulum is made from a mass m hanging from a (massless) string of length ℓ. The motion of the string is dictated by the acceleration g due to gravity. Use dimensional analysis to find the relevant time scale τ for the pendulum motion. Compare this to what you know to be the period of a pendulum undergoing small amplitude oscillations.

PHY2502 Mathematical Physics Quiz #2 Solution

A pendulum is made from a mass m hanging from a (massless) string of length ℓ. The motion of the string is dictated by the acceleration g due to gravity. Use dimensional analysis to find the relevant time scale τ for the pendulum motion. Compare this to what you know to be the period of a pendulum undergoing small amplitude oscillations.

$$\tau = m^x \ell^y g^z$$

$$[\tau] = [m]^x [\ell]^y [g]^z$$

$$T = M^x L^y L^z T^{-2z} = M^x L^{y+z} T^{-2z}$$

Therefore $x = 0$, $y + z = 0$, $z = -1/2$, and so $y = -z = +1/2$. In other words

$$\tau = \ell^{1/2} g^{-1/2} = \sqrt{\frac{\ell}{g}}$$

This agrees with what you know from your introductory physics course, namely the period of a pendulum $T = 2\pi\sqrt{\ell/g}$. Dimensional analysis can't, of course, predict the factor of 2π.