PHYS2502 Mathematical Physics Lab #13 24 Apr 2023

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your code to the github *repository, and a PDF of your output to the* canvas *page for the course.*

This lab is about "throwing dice" and showing that the probability of getting a certain number of "ones" follows the appropriate probability distribution(s). Instead of throwing actual dice, though, you will be using the random number generator in MATHEMATICA. See Figure 9.1 in the textbook, along with the associated text.

Imagine that you are throwing a handful of n dice. Each of them will land face up showing a number of dots between one and six. If you throw this handful m times, and each time count the number of dice i, $0 \leq i \leq n$, that land with a "one" facing up, then you'll have a measure of the probability of getting i ones on any particular throw.

You can do this experiment in MATHEMATICA using the RandomInteger function. Generate $n \times m$ random integers between 0 and 6, and then use Partition to break this up into m lists of n dice. You can get the list of the number of ones for each throw using

```
ones = Table[Length[Select[throws[[k]], # == 1 &]], {k, 1, m}];
```

Try this out first with a small number of throws, so that you can inspect them visually and check that you are getting the right number of ones. Ultimately, you want to do this with a large enough m so that even the least probable outcome has some events.

Use Histogram to make a plot of the frequency of ones. (I suggest lining up the bins so that their edges are on the half integers.) You should try using {"Log", "Count"} for the height specification, because the probability of getting all ones is so small, it will be difficult otherwise to see anything in that bin.

Now compare the histogram to the expected probability distribution. The probability for getting i ones in n chances (i.e. the number of dice in a throw) is given by

$$\mathcal{P}_{\text{bin}}(i) = \frac{n!}{i!(n-i)!} \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{n-i}$$

Plot the expected results on top of your histogram, using Show and ListPlot (or ListLogPlot).

You can also compare to the approximation based on $1/6 \ll 1$ and $n \gg 1$, namely

$$\mathcal{P}_{\text{Poiss}}(i) = e^{-\mu} \frac{\mu^i}{i!}$$

where $\mu = n/6$ is the expected average for the number of times that you get a one. For this comparison, you want to make sure that you make a log plot, and compare the two distributions, and your histogram, for the larger values of i where the probability is smallest.