PHYS2502 Mathematical Physics Lab #9 27 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your code to the github *repository, and a PDF of your output to the* canvas *page for the course.*

We learned in class that any solution $u(x,t)$ to the wave equation can be written as $f(x-vt)$ or $g(x+vt)$, where v is the speed of the wave, and $f(z)$ and $g(z)$ are arbitrary functions, or any linear combination of them.

Now consider a semi-infinite long stretched string, extending from its end at $x = 0$ to $x \to +\infty$. The end can either be fixed at $u(0,t) = 0$, or the end can float, in which case the string stays flat at $x = 0$, that is $\partial u/\partial x|_{x=0} = 0$. You should have no trouble convincing yourself that the solutions to the wave equation for these two cases are

$$\begin{aligned} u(x,t)_{\text{Fixed}} &= f(x+vt) - f(-x+vt) \\ \text{and} \qquad u(x,t)_{\text{Float}} &= f(x+vt) + f(-x+vt) \end{aligned}$$

Your task in this lab is to animate each of these two solutions, as a plot of the wave form as a function of x for $x \geq 0$. The region $x < 0$ is "unphysical", but you can include this region in your animation if you'd like. I think the result looks cooler if you just do $x \geq 0$, but you might find it insightful to include the unphysical region in your plots.

I suggest that you pick a form of $f(z)$ that is a "localized pulse." In my solution I just made $f(z)$ an isoceles triangle peaking at $z = 0$, which is easy enough to form with the Piecewise function or the HeavisideLambda function in MATHEMATICA. Other possibilities are a rectangular box, which you can make with Piecewise or HeavisidePi, or something in the shape of a Gaussian, namely

$$f(z) = Ae^{-z^2/2\sigma^2}$$

where σ is the "width", or something else that suits your fancy. All of my examples can be made to be symmetric about $z = 0$, but that is not necessary.

If you choose something centered on $z = 0$, then it is best to animate your plot over a time range $-T \leq t \leq T$ where $T = n\sigma/v$ where σ is some measure of the width of the pulse, and n is some factor like "a few." This way, you will see the pulse start somewhere on the positive x-axis, move to the left, reflect off the end, and then come back to where it started.

Create animations for both the fixed endpoint and floating endpoint solutions, and what your result when the pulse reflects to confirm that this boundary condition is in fact satisfied.