PHYS2502 Mathematical Physics Lab #8 20 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your code to the github *repository, and a PDF of your output to the* canvas *page for the course.*

This lab solves a partial differential equation, both analytically and numerically, and is likely best done in MATHEMATICA. The PDE you will be solving is the *Wave Equation* (in one spatial dimension x) which we will derive in class this week. This lab concerns itself with "standing waves", such as what you get on a stretched string with the ends fixed. (Think a guitar or violin string.) The Wave Equation is

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$$

where $u(x,t)$ is the shape of the string as a function of x, at some time t. The parameter v is called the "speed" of the wave, for reasons that you'll see when we cover this in class.

For each of the following solutions, assuming the string is fixed to $u = 0$ at $x = 0$ and $x = L$, and that the string is initially at rest, that is $\partial u/\partial t|_{t=0} = 0$. For DSolve or NDSolve in MATHEMATICA, you can write this initial condition as Derivative[0, 1][u][x, 0] == 0.

(a) Solve the Wave Equation for an initial shape $u(x,0) = A\sin(\pi x/L)$. This is known as the "fundamental mode". Animate your solution for some values of A, L, and v. Run the animation over several units of the time scale L/v.

(b) Repeat this for an initial shape $u(x,0) = A\sin(2\pi x/L)$. This is known as the "first harmonic". What obvious thing do you notice about the frequency of the oscillations?

(c) Make an animation of the sum of the solutions for (a) and (b). The result should remind you a little of the work we did on the coupled oscillations of two masses.

(d) Now consider an initial shape $u(x,0) = Ax^2(L-x)^2$. You can solve this with DSolve, but the result will look very odd to you. (You'll understand why after class this week.) Instead, you can solve the Wave Equation numerically with NDSolve. (Just make sure that you put in numbers for A, L, and v before you try to solve the equation.) Animate this solution, and see if you can identify a connection between the shapes and frequencies that you saw in (a) and (b).