PHYS2502 Mathematical Physics Lab #7 13 Mar 2022

This lab assignment is at 8am, the morning after the date shown, although you should able to complete it easily before the end of the lab period. When you're done, upload your code to the github *repository, and a PDF of your output to the* canvas *page for the course.*

This lab makes plots of functions of two variables, and is likely best done in MATHEMATICA. I will give you hints using MATHEMATICA functions, but you are welcome to use other plotting packages.

The lab has three parts. As always, ask yourself if your result makes sense (or not).

(a) The following function makes for a nice example of how to picture the gradient:

$$f(x,y) = y\frac{e^{-x}+1}{e^{-x}+3}$$

Make a ContourPlot of this function over some range of x and y. (I suggest you use the same ranges for each, and make them symmetric about the origin.) Choose contour values that are equally spaced and span an appropriate range given your ranges for x and y, and label the contours. In MATHEMATICA, I think it makes for a clearer plot if you turn off shading, but you are welcome to play around with the many options.

Now form the (two dimensional) vector field $\vec{v}(x,y) = \vec{\nabla} f$. Plot the vector field on top of the contour plot, showing the direction and magnitude of the field at each position plotted. In MATHEMATICA, use VectorPlot for this (and Show to combine the plots). Include a legend that shows the magnitudes of the vectors.

Finally, calculate $\vec{\nabla} \times \vec{v}$. (You know what the answer has to be, right?) To do this in MATHEMATICA, you'll need to convert $\vec{v}$ into a three dimensional vector. There are different ways to do this.

(b) Now make a vector plot of the field

$$\vec{v}(x,y) = r(-y\,\hat{\imath} + x\,\hat{\jmath}) \qquad \text{where} \qquad r = (x^2+y^2)^{1/2}$$

Then calculate $\vec{w} = \vec{\nabla} \times \vec{v}$ – *Which direction does it have to point in?* – and make a contour plot of $|\vec{w}|$, combined with your vector plot.

Finally, calculate $\vec{\nabla} \cdot \vec{w}$. (You know what the answer has to be, right?)

(c) Now make a vector plot of the field

$$\vec{v}(x,y) = (-y\,\hat{\imath} + x\,\hat{\jmath})/r^2 \qquad \text{where} \qquad r = (x^2+y^2)^{1/2}$$

and then express $\vec{w} = \vec{\nabla} \times \vec{v}$ in its simplest form. What do you think about this result?