PHYS2502 Mathematical Physics Homework #13 Due 25 Apr 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

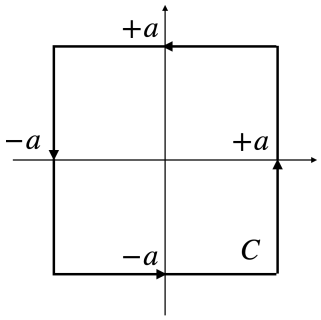
(1) If $z = x + iy$, where x and y are real numbers, then prove that the function $f(z) = e^z$ is analytic everywhere in the complex plane.

(2) If $z = x + iy$, where x and y are real numbers, then prove that the function $f(z) = 1/z$ is analytic everywhere in the complex plane except at $z = 0$.

(3) By direct integration, calculate the integral

$$\mathcal{I} = \int_C \frac{1}{z}\, dz$$

around the square contour with side length $2a$ shown here



and compare to the result you get from the Cauchy Integral Theorem.

(4) Evaluate the integral

$$\mathcal{I} = \int_{-\infty}^{\infty} \frac{e^{ikx}}{4x^2 + 1}\, dx$$

separately for the cases $k > 0$ and $k < 0$. Check your answers using MATHEMATICA.

(5) Consider two complex variables $w = u + iv$ and $z = x + iy$, and the "map" given by

$$w = z^2$$

For contours $u =$ constant in the w-plane, draw the contours to which they map in the z-plane. Repeat for contours $v =$ constant in the w-plane. Show that the two sets of contours in the z-plane are orthogonal to each other. That is, just as the contours for $u =$ constant and $v =$ constant are perpendicular to each other everywhere they intersect, so for the corresponding contours in the z-plane. For this reason, we refer to this mapping function as a "conformal map." Conformal maps have many applications in science and engineering.