PHYS2502 Mathematical Physics Homework #10 Due 4 Apr 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Write out the three equations for x, y, and z represented by

$$\underline{\underline{A}}\,\underline{X} = \underline{C} \qquad \text{where} \qquad \underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

and solve them for x, y, and z. Then determine the matrix $\underline{\underline{A}}^{-1}$ by writing your answer as $\underline{X} = \underline{\underline{A}}^{-1}\underline{C}$. You might want to check your answer using MATHEMATICA.

(2) Find the inverse matrix $\underline{\underline{A}}^{-1}$ for the matrix $\underline{\underline{A}}$ in Problem (1) by calculating the determinant using an expansion in minors, and then forming the matrix of cofactors.

(3) Construct a 3×3 matrix that rotates a three-dimensional vector $\underline{v}$ through an angle θ about the x-axis, combined with a reflection about the yz-plane, that is, takes x to $-x$. Pick two specific examples for $\underline{v}$ and show that your matrix does what it is supposed to do.

(4) Prove that if $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{0}}$ for two matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$, then the determinant of at least one of them must be zero. Find an example, however, of two 3×3 matrices that are each nonzero, and in fact do not have any full rows or columns with all zeros, but whose product $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{0}}$.

(5) A Lorentz transformation tells you how to convert space and time between two reference frames, call them the "primed" and "unprimed" frames, moving at a velocity v relative to each other, in accordance with the framework of Special Relativity. For a reference frame moving in the x-direction with respect to another frame, the Lorentz transformation is

$$x' = \gamma(x - vt) \qquad \text{and} \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$, with c being the speed of light. Then define a vector

$$\underline{x} = \begin{bmatrix} ct \\ x \end{bmatrix}$$

(a) Show that the transformation maintains the value of $s^2 = (ct)^2 - x^2$. (We say that the Lorentz transformation maintains the norm of a vector with a "Minkowski metric," instead of a "Euclidean metric.")

(b) Find the Lorentz transformation matrix $\underline{\underline{\Lambda}}$ which takes you from the unprimed frame to the primed frame, by acting on $\underline{x}$.

(c) Write $\underline{\underline{\Lambda}}$ in terms of a single parameter η which combines γ and β. Compare this to a rotation matrix in two dimensions.

(d) Show that the inverse transformation $\underline{\underline{\Lambda}}^{-1}$ corresponds to $v \to -v$ or, equivalently, to the change η to $-\eta$.