PHYS2502 Mathematical Physics   Homework #7   Due 14 Mar 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Find the equation of a plane which contains the point $(x, y, z) = (1, -2, 5)$ and which is perpendicular to a vector pointing from the origin into the first quadrant and which makes equal angles with the x, y, and z axes. Write your equation in the form $Ax + By + Cz = D$, where A, B, C, and D have numerical values.

(2) A line in space passes through the origin and is at an angle of $45°$ with respect to the positive z-axis and is at equal angles with respect to the positive x- and y-axes. Find the coordinates of the intersection point of this line with the plane in Problem **(1)** above.

(3) For spatial vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, prove that

$$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C})$$

I think the easiest way to do this is to write the vectors in terms of their components and make use of the Kronecker δ and the Levi-Civita symbol, and their properties.

(4) Use the result from Problem **(3)** to find an expression for $|\vec{A} \times \vec{B}|^2$ in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and their dot products. Explicitly show that this is the same as the geometric definition of the magnitude of the cross product.

(5) For a particle of mass m moving in a plane located at position $\vec{r}(t)$, find an expression for the kinetic energy

$$K = \frac{1}{2} m \left(\frac{d\vec{r}}{dt} \right)^2$$

in terms of plane polar coordinates r and ϕ. Do this explicitly by writing $\vec{r}$ first in terms of Cartesian coordinates x and y, convert to polar coordinates, and then take derivatives.