PHYS2502 Mathematical Physics Homework #6 Due 28 Feb 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

(1) Starting from the infinite power series expression for the Bessel function $J_m(x)$, where m is an integer, prove that

$$\frac{d}{dx}\left[x^m J_m(x)\right] = x^m J_{m-1}(x)$$

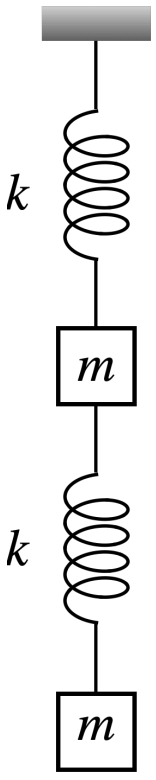
(2) Prove that the Legendre polynomials are "orthogonal", that is $\int_{-1}^{1} P_\ell(x) P_m(x)\, dx = 0$ if $\ell \neq m$. You can do this by writing down the differential equation for $P_\ell(x)$ and multiplying through by $P_m(x)$. Then create a second equation by reversing the indices, subtract the two equations and then integrate.

(3) It is possible to prove that $\int_{-1}^{1} P_\ell(x) P_\ell(x)\, dx = 2/(2\ell+1)$. (But we won't try to do that now.) Use this, along with the orthogonality of Legendre polynomials, to find an expression for the coefficients a_n in the expansion

$$f(x) = \sum_{m=0}^{\infty} a_m P_m(x)$$

where $f(x)$ is defined for $-1 \leq x \leq 1$. You can do this by multiplying both sides of this expression by $P_\ell(x)$ and integrating. Now use this find the first few nonzero coefficients for $f(x) = \sin(\pi x)$ and make a plot of the expansion compared to $f(x)$. (Doing the integrals and making the plots is much easier in MATHEMATICA than by hand.)

(4) Two identical mass hang vertically under their own weight from two identical springs from a fixed point on the ceiling, as shown in the figure on the right. Find the two normal frequencies and describe the amplitudes of the two normal modes.



(5) Three identical capacitors C are connected to two identical inductors L as shown in the figure below. Find two coupled differential equations for $q_1(t)$ and $q_2(t)$ and find the normal mode frequencies. Analyze the problem by equating the potential differences for legs 1, 2, and 3 between nodes A and B. Use the sign convention shown for the currents in each of the three legs which implies that $i_1 + i_2 + i_3 = 0$. You can assume the charges are all zero when the currents are all zero.