PHYS2502 Mathematical Physics Homework #5 Due 21 Feb 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

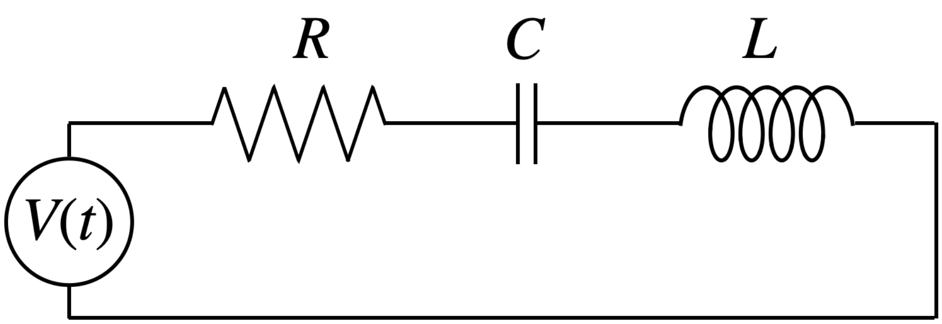
(1) A mass m moves in one dimension $x(t)$ connected to a spring with stiffness k, and is driven by a force term $F = ma\cos\omega t$ where a and ω are constants. Write down and solve the differential equation for $x(t)$ in terms of $\omega_0^2 \equiv k/m$ for the initial conditions $x(0) = \dot{x}(0) = 0$. Use a trigonometric identity to cast your solution in the form of a single product of sines. Write $\omega_0 - \omega = \epsilon$ with $|\epsilon| \ll \omega_0$, and describe the motion for short times $t \ll 1/|\epsilon|$ (but $t \gg 1/\omega_0$) and long times $t \gg 1/|\epsilon|$.

(2) An mechanical oscillator has position $x(t)$ governed by the equations

$$\ddot{x}(t) + 2\dot{x}(t) + 5x(t) = e^{-t}\cos(3t) \qquad x(0) = 0 \qquad \dot{x}(0) = 0$$

Find the motion $x(t)$ and plot it for $0 \le t \le 2\pi$. (You can use MATHEMATICA to handle the algebra, if you like, but I want you to solve the differential equation by hand.)

(3) The diagram below is of an electrical circuit with a resistor R, capacitor C, inductor L, and an AC voltage source $V(t)$ connected in series:



The voltage drop across the capacitor is q/C where $q(t)$ is the charge on the capacitor, the voltage drop across the resistor is iR where $i = dq/dt$ is the current in the circuit, and the voltage drop across the inductor is $L\,di/dt$. Kirchoff's Law says that the sum of all voltage drops around a closed path must be zero. If $V(t) = -V_0\cos(\omega t)$, then find $q(t)$ assuming that $q(0) = 0$ and $i(0) = 0$. You are welcome to quote directly from the solution we derived in class for the driven mechanical oscillator.

(4) Find the general solution $y(x)$ for the differential equation $y''(x) = y$ using the series solution approach, about $x = 0$, written as a linear combination of two separate infinite series. Show that that two series are in fact those for $\cosh(x)$ and $\sinh(x)$.

(5) Find a series solution for $y(x)$ about $x = 0$ for the differential equation

$$y'' - 2xy' + \lambda y = 0$$

in terms of two independent series solutions $y_0(x)$ and $y_1(x)$. For what values of λ is the solution a polynomial? Find the polynomial solution for $\lambda = 4$.