# PHYS2502 Mathematical Physics Homework #2 Due 31 Jan 2023

*This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.*

**(1)** The motion of a damped harmonic oscillator in one dimension is given by

$$x(t) = Ae^{-\beta t}\cos(\omega t + \phi)$$

Find $A$ and $\phi$ in terms of the initial conditions $x(0) = x_0$ and $v(0) = v_0$. Assume that $A$, $\beta$, and $\omega$ are all real and positive. (You are welcome to solve this in MATHEMATICA, but in this case submit a PDF of your solution notebook.)

**(2)** Consider a straight rod of length $\ell$ and mass $m$. The center of mass of the rod is

$$x_{\text{CM}} = \frac{1}{m}\int_0^L x\, dm$$

where $x$ measures the position along the rod.

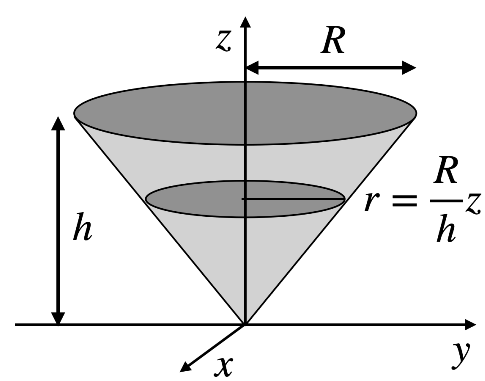
(a) Show that $x_{\text{CM}}$ is what you expect if the rod has uniform mass density.

(b) Now calculate $x_{\text{CM}}$ assuming that the mass density $\lambda(x)$ of the rod grows linearly from zero at the end of the rod at $x = 0$. Express your answer as a constant times $L$.

**(3)** The figure shows an inverted vertical right circular cone of uniform mass density and height $h$ and base radius $R$, with symmetry around the $z$-axis. The moment of inertia for an object $\mathcal{O}$ with mass $m$ is given by

$$I = \int_{\mathcal{O}} (x^2 + y^2)\, dm = \int_{\mathcal{O}} \xi^2\, dm$$

where $\xi = (x^2+y^2)^{1/2}$ is the distance from the $z$-axis for an infinitesimal mass element $dm$. Find the moment of inertia of the cone in terms of $m$, $h$, and $R$. You might start by finding the moment of inertia of a disk with radius $r$ and thickness $dz$.




**(4)** Use the definitions of hyperbolic sine and hyperbolic cosine in terms of exponential functions to prove that

$$\sinh(x + y) = \sinh(x)\cosh(y) + \sinh(y)\cosh(x)$$

**(5)** Evaluate the following integral

$$\int_0^\infty x^4 e^{-ax^2}\, dx$$

using the techniques described in Section 1.5.6. This integral is used to find the root-mean-square velocity of gas particles that follow the Maxell-Boltzmann Distribution in statistical mechanics.