PHYS2502 Mathematical Physics Homework #1 Due 24 Jan 2023

This homework assignment is due at the start of class on the date shown. Please submit a PDF of your solutions to the Canvas page for the course.

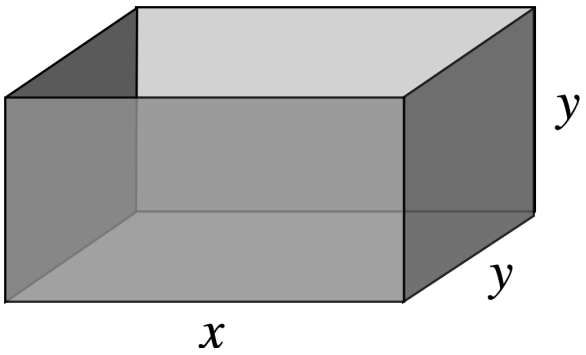
(1) The Hubble constant H_0 measures the expansion rate of the universe. It's value has been measured to be 70 km/sec per Mpc, where one megaparsec (Mpc) is a common measure of cosmological distances, and equals 3.1×10^{19} km. If H_0 is truly a constant in time, then what is the age of the universe? Express your answer in years.

(2) Use dimensional analysis to estimate the energy of an electron with mass m bound in an atom with size $a = 10^{-10}$ m. In this case, the scale is set quantum mechanically according to the quantity $\hbar$, which has the same dimensions as angular momentum. Express your answer in electron volts. (When working in quantum mechanics, it is handy to remember that $\hbar c = 200$ MeV·fm and that $mc^2 = 0.511$ MeV for an electron.)

(3) Use dimensional analysis to find an expression for the pressure at the center of the Sun, assuming it only depends on gravity and the solar mass and radius. Now assume the Sun has uniform density, is made only of hydrogen, and follows the ideal gas law to find the temperature at the center of the Sun.

(4) The equation $ax^2+by^2 = c$, where a, b, and c are positive constants, describes a collection of points (x, y) that lie on an ellipse. Find the two points at which the slope $dy/dx = 1$ in terms of a, b, and c.

(5) You have a fixed number of square feet of lumber with which to build an open box of maximum volume. The box must have square sides, and no top:



Find the ratio of the length of the base to the height of the box.