Correction

Eric King - Jan 31st Class

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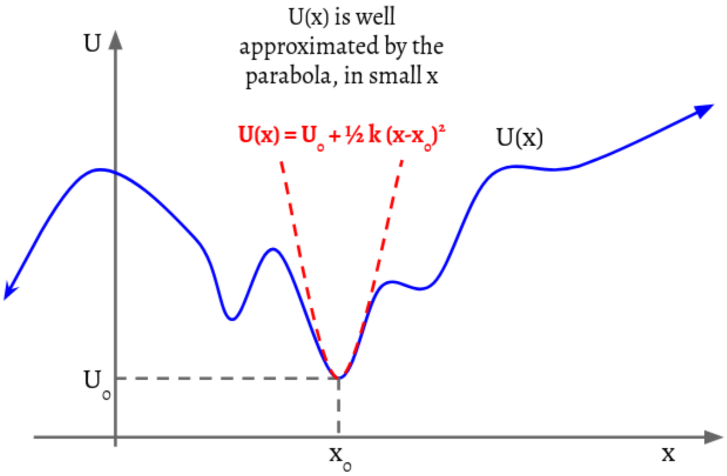

1 End of class mistake

My apologies as I realized this morning that I wrote the last in-class example incorrectly.

This is what I wrote (which is incorrect):

$$U(x) \approx U(x-x_0) + U'(x-x_0)(x) + \frac{1}{2!}U''(x-x_0)(x)^2$$

Expanding the arbitrary potential around point x_0 we should evaluate derivatives at $x = x_0$ and expand the summation around powers of $(x - x_0)$.

$$U(x) = U(x_0) + U'(x_0)(x-x_0) + \frac{1}{2!}U''(x_0)(x-x_0)^2 + \ldots$$

We established that the first derivative is zero since the potential at x_0 is a local minimum and that the second derivative must be positive. Under the condition that $x << 1$ we can truncate at second order.

$$U(x) \approx U(x_0) + \underbrace{U'(x_0)}_{\text{Evaluates to 0}}(x-x_0) + \frac{1}{2!}\underbrace{U''(x_0)}_{\text{Evaluates to `k'}}(x-x_0)^2$$

$$U(x) \approx U(x_0) + \frac{1}{2}k(x-x_0)^2$$

From here everything is the same:

$$F = -\frac{dU}{dx} = -k(x-x_0) \quad \text{[Spring Force]}$$

The mechanical behavior of this situation should be familiar from Physics I. Therefore, within small deviations in x, a particle trapped in a potential well acts like an oscillator.