PHYS2063 Wave Physics Homework #13 Due Tuesday 11 Oct 2022

This homework assignment is due at the start of class on the date shown. You may submit a PDF of your solutions to the Canvas page for the course, or bring a paper copy to class.

(1) In addition to Gauss' Theorem and Stokes' Theorem, there is a third "surface theorem" which, for some reason, doesn't find its way much into physics problems. The theorem is

$$\int_V \vec{\nabla} f \, dV = \oint_S f \, d\vec{S}$$

where $f(x, y, x)$ is some scalar field, and S is the surface enclosing V. Prove this using Gauss' Theorem and the vector field $\vec{A} = \vec{C}\, f$ where $\vec{C}$ is some arbitrary constant vector.

(2) Show that Gauss' Theorem holds for the vector field $\vec{A} = \hat{i}x + \hat{j}y + \hat{k}z$ and the cubic volume V with side length L in the first octant (x, y, and z all positive) with one corner at the origin.

(3) Show that Stokes' Theorem holds for the vector field $\vec{A} = -\hat{i}y + \hat{j}x$ and the square surface S with side length L in the first quadrant (x and y both positive) with one corner at the origin.

(4) Two homework assignments ago, you calculated the divergence of the vector field

$$\vec{E}(x, y, z) = \frac{\hat{i}\, x + \hat{j}\, y + \hat{k}\, z}{(x^2 + y^2 + z^2)^{3/2}} = \frac{1}{r^2}\hat{r}$$

where the second form just uses the definition of the position vector $\vec{r} = \hat{i}\, x + \hat{j}\, y + \hat{k}\, z$ and the radial unit vector $\hat{r} = \vec{r}/r$. Use your result for the divergence to test Gauss' Theorem for the spherical volume V bounded by the spherical surface S with radius R, centered on the origin. The surface integral is very easy to calculate since $r = R$ on this surface, and the unit vector $\hat{r}$ is perpendicular to the surface everywhere. However, the result will look like a violation of Gauss' Theorem. Can you see what is the source of the problem? (That's a pun, by the way.) You might recognize that $\vec{E}$ is proportional to the electric field from a point charge.