PHYS2063 Wave Physics   Homework #4   Due Tuesday 6 Sep 2022

This homework assignment is due at the start of class on the date shown. You may submit a PDF of your solutions to the Canvas page for the course, or bring a paper copy to class.

(1) We derived a differential equation in class for the damped oscillator, namely

$$\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x(t) = 0$$

Consider the "critical damping" case where $\beta = \omega_0$. Substitute $x(t) = u(t)e^{-\beta t}$ to find, and then solve, a differential equation for $u(t)$. Apply the initial conditions to find the complete solution for $x(0) = x_0$ and $\dot{x}(0) = v_0$.

(2) Make plots of the motion $x(t)$ for the damped oscillator for the initial conditions $x_0 = 2$ and $v_0 = 0$, for each of the three cases

(a) $\beta = \omega_0/5$
(b) $\beta = 5\omega_0$
(c) $\beta = \omega_0$

Plot as a function of the time t in units of $2\pi/\omega_0$.

(3) Consider a (linear) lightly damped mechanical oscillator with mass m, spring constant k, and damping coefficient b, where $b/m \ll \sqrt{k/m}$. Find an expression for the fractional decrease in energy over one period of oscillation, in terms of m, k, and b. (It is probably easiest to do this by considering how the energy depends on the amplitude, and then finding the fractional decrease in the amplitude.)

Also express your answer in terms of $\beta = b/2m$ and $\omega_0^2 = k/m$.

The inverse of this fraction is often written as $Q/2\pi$ where Q is called the *Quality Factor*.

(4) Find an expression for Q in terms of L, C, and R for the series LCR circuit in the previous homework assignment.