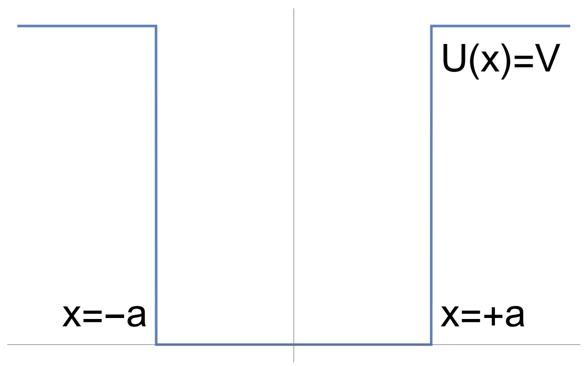
PHYS2063 Wave Physics Fall 2017 HW #12 Due 8 Dec 2017

(1) A wave function $\psi(x)$ satisfies the Schrödinger equation

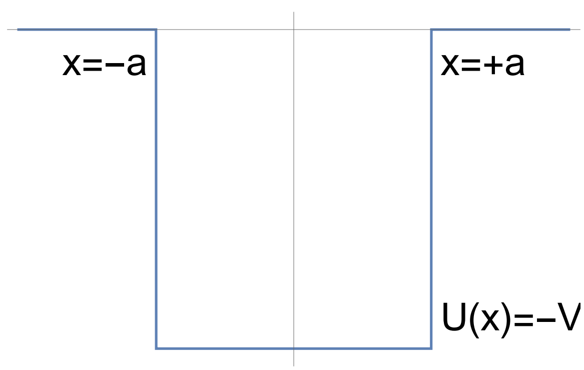
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$$

where the potential energy function $U(x)$ is symmetric, that is $U(-x) = U(x)$. Show that either $\psi(-x) = +\psi(x)$, i.e. the wave function has "positive parity", or $\psi(-x) = -\psi(x)$, i.e. "negative parity." (Note that $\psi(-(-x)) = \psi(x)$.) Use this to find all the the bound state eigenvalues and eigenfunctions for the symmetric finite well



where $V = 5\hbar^2/ma^2$. Normalize the wave functions, plot them, and calculate the probability that the particle is found inside the wall, that is, within $-a \leqslant x \leqslant a$. You can follow the Mathematica example we did in class, but be careful of the different boundaries.

(2) A particle with mass m and energy $E = \hbar^2k^2/2m$ is incident from the left onto the square well with depth V shown below.



Show that the transmission probability T is given by

$$\frac{1}{T} = 1 + \frac{1}{4}\frac{V^2}{E(E+V)}\sin^2 2\kappa a \qquad \text{where} \qquad \frac{\hbar^2\kappa^2}{2m} = E + V$$

Explain the physical conditions under which $T = 1$.

Note: This calculations explains the apparently anomalous phenomenon of low energy scattering of electrons from atoms, known as the Ramsauer Effect.