PHYS2063 Wave Physics Fall 2017 HW #9 Due 10 Nov 2017

(1) In class we showed that any function of the form

$$w(x,y,z,t) = f(\mathbf{k}\cdot\mathbf{r} - \omega t) = f(k_x x + k_y y + k_z z - \omega t)$$

was a solution to the three dimensional wave equation, that is

$$\boldsymbol{\nabla}^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2}$$

so long as $\omega = |\mathbf{k}|c$. This form is called a *plane wave*. Now show that a different functional form, namely

$$w(x,y,z,t) = \frac{f(kr - \omega t)}{r} \qquad \text{where} \qquad r \equiv \sqrt{x^2+y^2+z^2}$$

is a solution to the same wave equation, so long as $\omega = kc$.. This is called a *spherical wave* and would be used to satisfy a set of boundary conditions much different than those used for a plane wave.

(2) A plane parallel plate of glass of thickness d has a non-uniform refractive index n given by $n = n_0 - \alpha r^2$ where n_0 and α are constants and r is the distance from a certain line perpendicular to the sides of the plate. Show that this plate behaves as a converging lens of focal length $1/2\alpha d$.