PHYS2063 Wave Physics Fall 2017 HW #8 Due 27 Oct 2017

(1) A function $f(x)$ is zero for $x \leqslant -\alpha$ and $x \geqslant \alpha$, but for $-\alpha \leqslant x \leqslant \alpha$,

$$f(x) = \frac{1}{\alpha^5}(x-\alpha)^2(x+\alpha)^2$$

Plot the function and explain why 2α is a good measure of its width. Then find the Fourier transform $a(k)$ and plot it, and use the points nearest $k = 0$ to define its width. Show that the product of the two widths is independent of α.

Extra Credit (Double points): If you normalize the functions $f(x)$ and $a(k)$ so that their integrals equal unity, you can interpret them as probability distributions. This lets you define formal "widths" given by

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad \text{and} \qquad \Delta k = \sqrt{\langle k^2 \rangle - \langle k \rangle^2}$$

where $\langle q \rangle$ means the average of quantity q, which you get by averaging it over the corresponding probability distribution. Use this to find Δx and Δk. Find the product of these and show that the result is independent of α.

(2) The "step function" $H(x)$ is defined so that $H(x) = 0$ for $x < 0$, and $H(x) = 1$ for $x > 0$. Show that dH/dx has the correct properties to claim that $dH/dx = \delta(x)$.