This assignment has only one problem, but it will count double.

We discussed in class the problem two identical masses coupled with three identical springs:



(a) Find the solutions for  $x_1(t)$  and  $x_2(t)$  given the general initial conditions  $x_1(0) = x_{1_0}$ ,  $x_2(0) = x_{2_0}$ ,  $\dot{x}_1(0) = v_{1_0}$ , and  $\dot{x}_2(0) = v_{2_0}$ . You are welcome to follow the ansatz solution we developed in class, or to use MATHEMATICA to solve the differential equations directly. (You can use matrix techniques if you're really ambitious, but that is not necessary.)

(b) Apply the initial conditions  $x_{1_0} = A$  and  $x_{2_0} = v_{1_0} = v_{2_0} = 0$  to show that you get the same answer we came up with in class.

(c) Show that the combinations  $x_{\text{Plus}}(t) \equiv x_1(t) + x_2(t)$  and  $x_{\text{Minus}}(t) \equiv x_2(t) - x_1(t)$  reduce to expressions having a single frequency, for arbitrary initial conditions.

(d) Using the initial conditions  $x_{1_0} = 5$ ,  $x_{2_0} = 0$ ,  $v_{1_0} = -15$ , and  $v_{2_0} = 0$ , plot on one set of axes  $x_1(t)$  and  $x_2(t)$ , and on another set of axes  $x_{\text{Plus}}(t)$  and  $x_{\text{Minus}}(t)$ . Use  $\omega_0 = 2\pi$  so that the "base period"  $2\pi/\omega_0 = 1$ . It should be clear that  $x_{\text{Plus}}(t)$  and  $x_{\text{Minus}}(t)$  have the correct single period, while  $x_1(t)$  and  $x_2(t)$  do not have a single frequency.