PHYS2063 Wave Physics Fall 2017 HW #4 Due 22 Sep 2017

(1) The generic differential equation we use for damped harmonic motion is

$$\ddot{u} + 2r\dot{u} + \omega_0^2 u = 0$$

(a) Solve this equation to find $u(t)$ for $\omega_0 = 8r$, with initial conditions $u(0) = A$ and $\dot{u}(0) = 0$. Plot the solution for a few oscillation cycles, and include, on the same plot, the undamped solution $u(t) = A\cos\omega_0 t$. Choose whatever values you'd like for ω_0 and A.

(b) Now find $u(t)$ for $r = 4\omega_0$. Use $u(0) = A$ but find the value of $\dot{u}(0)$ for which $u(t) = 0$ when $t = 4/r$. Using whatever values of A and r you would like plot the result in such a way that you can see the zero crossing at $t = 4/r$ and that $u(t) \to 0$ at long enough times.

(c) Repeat (b) for $r = \omega_0$.

(2) For the case of very weak damping of a harmonic oscillator, use Taylor expansions to find the lowest order expressions, only in terms of the quality factor Q, for (a) the fractional energy loss in one cycle of the oscillator, and (b) the fractional difference between the oscillator frequency and the frequency for no damping. In each case, show that you get the correct limiting value in the $Q \to \infty$.