

(1) A mass m moves in one dimension x under the action of a (net) force $F(x)$. The force can be derived from a function $U(x)$ according to $F(x) = -dU/dx$. Show that the quantity

$$E \equiv \frac{1}{2}m\dot{x}^2 + U(x)$$

is a constant of the motion. That is, $dE/dt = 0$. What is the common name for $U(x)$? Find $U(x)$ for a spring force, $F(x) = -kx$.

(2) A mass m moves in one dimension x under the action of the sum of two forces. One is a spring with stiffness k . The other is a linear drag force that is directly proportional to its velocity, with proportionality constant $b > 0$. Show that the mechanical energy $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ decreases at a rate bv^2 where v is the velocity of the mass.