Temple University

Undergraduate Mathematics Lecture Series

Submajorization and the Geometry of Unordered Collections, with Applications to Music and Welfare Economics.

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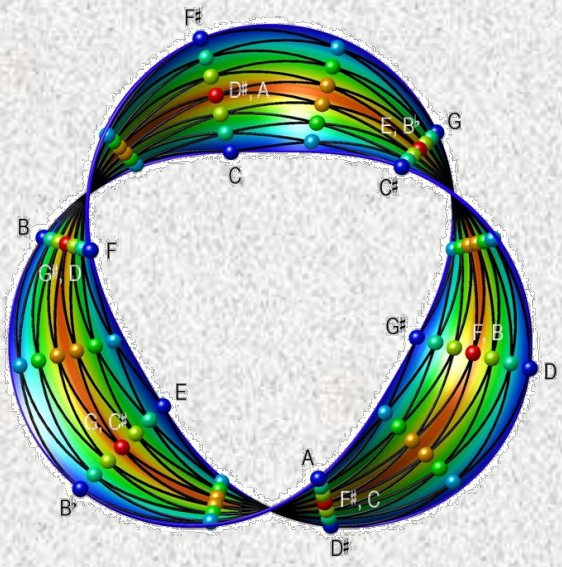
A musical orbifold: The set of two- and one-note chords forms a thrice-twisted Möbius strip, with the one-note chords on the boundary forming a trefoil knot.



February 24th, 1:00pm
Wachman Hall 617

If several voices or musical instruments sound a sequence of chords, our ears track the movement in each voice. This association is called voice leading. Although composers are normally instructed to "minimize" the overall amount of vocal movement, there is no principled way of choosing a measure for vocal movement when multiple individuals are involved. A similar problem arises in welfare economics: if we wish to measure income volatility, we must measure multiple income changes simultaneously. Tymoczko (2006) proposed requirements that every method of comparing voice leadings should satisfy. We show that these requirements are equivalent to the submajorization partial ordering, defined by Hardy, Littlewood, and Polya (1934) and developed in the study of welfare economics. We further show how to use submajorization to compare distances in the orbifolds (quotients