MATH 2121 **Mathematical Modeling and Simulation** **Spring 2025**

Problem Set 5

(Out Tue 03/11/2025, Due Tue 03/25/2025)

Submissions are to be done by sending an email with subject `MATH 2121: Problem set 5` to the course instructor, containing: all requested Matlab files (called `yourfamilyname_problem5X.m`), plus a single file (PDF preferred), called `yourfamilyname_pset5.pdf`, that contains all requested explanations.

Problem 5

Modify the Matlab file `temple_abm_traffic_car_following.m` from the course website `http://faculty.cst.temple.edu/~seibold/teaching/2025_2121/` in the following ways:

(a) Change the initial vehicle velocities to `v = (1:n)'`, and submit your code as `yourfamilyname_problem5a.m`. Present the results that happen; explain what the new initial state represents and why the observed behavior happens.

(b) Now change the time step `dt` to 0.1, and the number of compute steps per plotting event `np` to 1. This way, the same time as before passes between plotting events, but now with only one, rather than 10, time steps. Submit your code under the filename `yourfamilyname_problem5b.m`. Running the code should produce a weird behavior. Explain what happens and why.

(c) For the previous code, change the time stepping methods from Euler's method to Runge-Kutta 4. Submit your code under the filename `yourfamilyname_problem5c.m`, and explain why the new time stepping methods fixes the problem encountered in (b).

(d) Start with your code from part (c), change (back) to `dt = 1e-2` and `np = 10`, but keep the Runge-Kutte 4 time stepping. With this very accurate code, conduct a parameter study how the characteristic features of the arising traffic wave depend on the strength of the optimal velocity term. Specifically, vary the coefficient (which is 0.5 in the original file) in front of the term from 0 to 2, in increments of 0.05. For each choice, run the simulation to a sufficiently long final time and extract the maximum and minimum vehicle velocity, $u_{\max}$ and $u_{\min}$, at the final time. Using these data, plot $u_{\max}$ and $u_{\min}$ as functions of the parameter. Submit your code under the filename `yourfamilyname_problem5d.m`. Show the resulting plot, describe your observations, and explain the results.