Temple Math 8200 **Multiscale Modeling and Methods** **Fall 2024**

Problem Set 8 by Logan Reed

(Out Wed 11/13/2024, Due Wed 11/20/2024)

Problem 1

The Jacobi Schwarz algorithm is an iterative method for two overlapping domain decompositions which solves concurrently in all subdomains, $i = 1, 2$:

$$\begin{cases} -\Delta\left(u_i^{n+1}\right) = f & \text{in } \Omega_i \\ u_i^{n+1} = 0 & \text{on } \partial\Omega_i \cap \Omega \\ u_i^{n+1} = u_{3-i}^n & \text{on } \partial\Omega_i \cap \bar{\Omega}_{3-i} \end{cases}$$

The P.L. Lions' algorithm is an iterative method for two non-overlapping domain decompositions which solves concurrently in all subdomains, $i = 1, 2$:

$$\begin{cases} -\Delta\left(u_i^{n+1}\right) = f & \text{in } \Omega_i \\ u_i^{n+1} = 0 & \text{on } \partial\Omega_i \cap \Omega \\ \left(\frac{\partial}{\partial \mathbf{n}_i} + \alpha\right)\left(u_i^{n+1}\right) = \left(\frac{\partial}{\partial \mathbf{n}_i} + \alpha\right) u_{3-i}^n & \text{on } \partial\Omega_i \cap \bar{\Omega}_{3-i} \end{cases}$$

(a) Rewrite the boundary conditions and formulate a Finite Difference Method for both methods in the 1D case.

(b) Let $f(x) = (1 - \frac{1}{1+e^{-100(x-0.1)}})\sin(500\pi x)$. Solve the Poisson Equation over $[0, 1]$ using P.L. Lions with an interface at $x_0 = 0.05$ and $\alpha = 1$ and plot the results and residuals after 5 iterations. Find the pair (x_0, α) which maximizes the convergence rate.

(c) Let $f(x) = \sin(10\pi x)$ for $x \in [0, 0.5]$ and $f(x) = \sin(100\pi x)$ for $x \in [0.5, 1]$. Solve the Poisson Equation over $[0, 1]$ using Jacobi-Schwarz with interface centered at $x_0 = 0.55$ with an overlap width of $w = 0.2$ and plot the results and residuals after 5 iterations. Find the pair (x_0, w) which maximizes the convergence rate.

Instructions

Email your solutions (i.e., a scan or typed version of your pen-and-paper part; and programming codes in a way that they can be run by someone else) to `l.reed@temple.edu` with the email subject `Math 8200. Homework 8` and all the submitted filenames starting with your family name.