Temple Math 8200 **Multiscale Modeling and Methods** **Fall 2024**

Problem Set 7 by Youmna Layoun

(Out Wed 11/6/2024, Due Wed 11/13/2024)

Problem 1

This problem will guide you through implementing the Tree Code discussed in class using the Fast Multipole Method.

Given the set of 15 source particles:

```
X
   =[0.02,0.04,0.06,0.13,0.17,0.23,0.35,0.45,0.58,0.65,0.68,0.73,0.82,0.94,0.96];


q = [21,30,47,23,84,19,23,17,23,44,31,92,43,18,90];
q = q.*0.001;
```

where q(i) is the charge of particle X(i).

Let $y = 0.59$ and $G(x_j - y_i) = \frac{1}{|x_j - y_i|^2}$.

(a) Find the exact value of the sum $u = \sum_j G(x_j - y)q_j$.

(b) Find the numerical value of $u = \sum_j G(x_j - y)q_j$ according to the algorithm explained in class and the steps below.
Note: you can use `MATLAB` for faster evaluations.
Perform the following steps for $p = 1$ and again for $p = 2$:

(i) Setup the binary tree using $J = 3$ number of levels and identify the cell where y lies.

(ii) Find the interaction list of that cell which consists of the cell itself, the adjacent ones (near-field), and far-field cells such that they cover all of $[0, 1]$.

(iii) Calculate the weight of each far-field cell T in the interaction list using the formula discussed in class

$$w_m = \sum_{x_j \in T} q_j a_m(x_j - x^*)$$

for m from 0 to p, where x^* is the center of T.

(iv) Calculate the contribution of the far-field terms to the sum

$$\sum_{m=0}^{p} w_m S_m(x^* - y)$$

(v) Calculate the numerical value of u by adding near-field and far-field terms.

(c) For each value of p, compare the numerical value of u with the exact value by taking the relative error

$$\frac{|u_{\text{FMM}} - u_{\text{exact}}|}{|u_{\text{exact}}|}$$

Instructions

Email your solutions (i.e., a scan or typed version) to `tur61276@temple.edu` with the email subject `Math 8200. Homework 7` and all the submitted filenames starting with your family name.