Temple Math 8200 **Multiscale Modeling and Methods** **Fall 2024**

Problem Set 6 by Blessing Nwonu

(Out Wed 10/30/2024, Due Wed 11/6/2024)

Consider the following kinetic traffic models:

The Prigogine model

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -\frac{f - f_0}{T} + (1-P)f(t,x,v)\int_0^\infty (v'-v)f(t,x,v')dv' \tag{1}$$

The Paveri-Fontana model

$$\frac{\partial g}{\partial t} + v\frac{\partial g}{\partial x} = -\frac{\partial}{\partial v}\left(\frac{w-v}{T}g\right) + (1-P)\left[f(t,x,v)\int_v^\infty (v'-v)g(t,x,v',w)dv' - g(t,x,v,w)\int_0^v (v-v')f(t,x,v')dv'\right] \tag{2}$$

The reduced Paveri Fontana model

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -\frac{\partial}{\partial v}\left(\frac{V_0 - v}{T}f\right) + (1-P)f(t,x,v)\int_0^\infty (v'-v)f(t,x,v')dv' \tag{3}$$

Where, $f_0(t,x,v) = \rho(t,x)\tilde{f}_0(v)$ ($\tilde{f}_0(v)$ is a probability density function), $P = 1 - \frac{\rho}{\rho_{max}}$,$T = \tau\frac{1-P}{P}$ ($\tau > 0$), $V_0 = \omega(\rho)v$.

(a) Show that interactions are conservative in the Prigogine model (i.e., they preserve the total number of vehicles in the system).

(b) Consider the reduced Paveri-Fontana model together with the following boundary conditions:

$$\lim_{v\to 0} f(t,x,v) = 0$$

$$\lim_{v\to\infty} f(t,x,v) = 0$$

Show that in this model interactions are also conservative.

(c) Derive the traffic momentum equation for the reduced Paveri-Fontana model.

(d) Show that the desired speed distribution of the Paveri-Fontana model depends on the evolution of the system, (Unlike the Prigogine model, where the desired speed distribution is a priori). (Hint: Obtain a PDE for the desired velocity distribution function from the Paveri-Fontana model).

Instructions

Email your solutions (i.e., a scan or typed version of your pen-and-paper part) to `blessing.nwonu@temple.edu` with the email subject `Math 8200. Homework 6` and all the submitted filenames starting with your family name.