Temple Math 8200 **Multiscale Modeling and Methods** **Fall 2024**

Problem Set 4 by Afrina Asad Meghla

(Out Mon 10/14/2024, Due Mon 10/21/2024)

Problem 1

Consider the time-independent Schrödinger equation for a particle in a 1D potential well is given by:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where $V(x)$ is the potential, $\hbar$ is the reduced Planck's constant, m is the mass of the particle, and E is the energy eigenvalue.

(a)The potential for this system is:

$$V(x) = \begin{cases} 0, & -L < x < L \\ \infty, & \text{otherwise} \end{cases}$$

where L is the half width of the well.

Derive the equation for wavefunction and eigenvalue for the well for symmetric case.

Ans: The expected energy level(eigenvalue): $E_n = \frac{n^2\pi^2\hbar^2}{8mL^2}$

(b) Now Consider this potential for the system:

$$V(x) = \begin{cases} 0, & -L < x < L \\ V_0, & \text{otherwise} \end{cases}$$

Derive the wavefunctions considering they need to be continuous and smooth on the boundary for symmetric case.

Your expected general form of solution for eigenfunctions:

- $\psi_1 = Ce^{\alpha x}$ (while, $x < -L$)
- $\psi_2 = A\sin(kx) + B\cos(kx)$ (while, $-L < x < L$)
- $\psi_3 = De^{-\alpha x}$ (while, $x > L$)

Now plot the eigenvalues since in this case they cannot be solved analytically.

Hint:Take $u = \alpha L, v = kL$ and plot for $u^2 = u_0^2 - v^2$ and $u = v\tan(v)$

(c) If $u_0 \to \infty$, what can you say about energy level from your solution?

Instructions

Email your solutions (i.e., a scan or typed version of your pen-and-paper part; and programming codes in a way that they can be run by someone else) to `tuq07829@temple.edu` with the email subject `Math 8200. Homework 4` and all the submitted filenames starting with your family name.