Temple Math 8200 **Multiscale Modeling and Methods** **Fall 2024**

Problem Set 2 by Jacob Woods

(Out Mon 09/30/2024, Due Mon 10/7/2024)

Problem 1

For each of the following examples of wait time and jump length PDF Fourier or Laplace space asymptotic behavior, derive the mean squared displacement

$$< x^2(t) >= \int_{-\infty}^{\infty} x^2 p(x,t) dx \tag{1}$$

or show it diverges for long times. Use the Montroll–Weiss formula:

$$\hat{\bar{p}}(k,u) = \frac{1-\bar{w}(u)}{u} \frac{1}{1-\bar{w}(u)\hat{\lambda}(k)} \tag{2}$$

where $\bar{w}(u)$ is the Laplace transform of the wait time PDF and $\hat{\lambda}(k)$ is the Fourier transform of the jump length PDF.(HINT: look at the formula for $\hat{\bar{p}}(k,u)$ in terms of $\hat{p}(x,u)$)

(a)

$$\begin{aligned} \hat{\lambda}(k) &\sim 1 - \sigma^2 k^2 \\ \bar{w}(u) &\sim 1 - \tau u \ . \end{aligned} \tag{3}$$

(b) let $0 < \alpha < 1$ (subdiffusion)

$$\begin{aligned} \hat{\lambda}(k) &\sim 1 - \sigma^2 k^2 \\ \bar{w}(u) &\sim 1 - (\tau u)^\alpha \ . \end{aligned} \tag{4}$$

(c) let $1 < \beta < 2$ (Levy flights)

$$\begin{aligned} \hat{\lambda}(k) &\sim 1 - (\sigma k)^\beta \\ \bar{w}(u) &\sim 1 - \tau u \ . \end{aligned} \tag{5}$$

Problem 2

Simulate random walks for $t \in [0, 1000]$ to confirm the mean squared displacement scaling as a function of time for subdiffusion. For the step length and wait time PDF use:

$$w(x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{\alpha-1}{x^\alpha}, & \text{if } x \geq 1 \end{cases} \tag{6}$$

$$\lambda(t) = \frac{1}{2\pi} e^{\frac{-x^2}{2}} \ . \tag{7}$$

(a) For $\alpha = 3$ conduct a convergence experiment or give an argument to decide how many random walks are sufficient to recover the expected slope for the mean squared displacement as a function of time when plotted in log-log scale.(slope=1)

(b) Produce a plot for a sufficiently large number of random walks for $\alpha = 1.25, 1.5, 1.75, 2, 3$ to demonstrate the relationship between mean squared displacement and time in question 1. Explain why in the case of $\alpha = 2$ vs $\alpha = 3$ this relationship looks similar.

Instructions

Email your solutions (i.e., a scan or typed version of your pen-and-paper part; and programming codes in a way that they can be run by someone else) to `tuf98099@temple.edu` with the email subject `Math 8200. Homework 2` and all the submitted filenames starting with your family name.