Traffic flow analysis / modeling

Microscopic Scale

→ each vehicle is an individual entity

a) an ODE can be written for each tracking change in position & velocity

b) cellular automaton model
Nagel-Schreckenberg model

Macroscopic scale

Use systems of PDEs to track quantities like the density of vehicles or mean velocity

Think of our discussion on diffusion.

<u>Terms</u>

Density (k) → number of vehicles per unit length of roadway

Flow (q) → number of vehicles passing a reference point per unit of time.

Inverse of flow is headway (h) → time that elapses between i^{th} vehicle passing a reference point in space and the $(i+1)^{th}$ vehicle

In congestion → h is constant
as traffic jam forms $h \to$ infinity

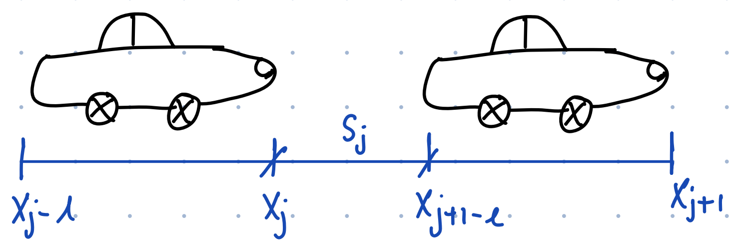
$q = kv \quad , \quad q = \frac{1}{h}$

Car following Models — Agent based implementation

$$\ddot{x}_j = F(s_j, \dot{x}_j, \dot{x}_{j+1})$$

x_j position
$\dot{x}_j$ speed
$\ddot{x}_j$ acceleration
$\dot{x}_{j+1}$ lead vehicle speed

$s_j = x_{j+1} - x_j - l$
gap to leader



Two models

A) **follow the leader model**

equilibrium flow stable

$$\ddot{x}_j = \beta \frac{\dot{x}_{j+1} - \dot{x}_j}{(s_j)^2}$$

acceleration proportional to difference in speed and inversely proportional to the square of the difference in position

if $\dot{x}_{j+1} > \dot{x}_j$ lead vehicle faster
$\dot{x}_{j+1} - \dot{x}_j > 0$ so acceleration positive

$\dot{x}_{j+1} < \dot{x}_j$ lead vehicle slower → slow down
acceleration negative
decceleration

β proportionality constant m^2/s

equilibrate speed to speed of lead vehicle

rapid acceleration if s_j small
(s_j = gap to lead vehicle)

less rapid if s_j large

Speed equilibrate rapidly when s_j small

B) More general

$$\ddot{x}_j = \beta \frac{\dot{x}_{j+1} - \dot{x}_j}{(s_j)^2} + \alpha\left(V(s_j) - \dot{x}_j\right)$$

may be unstable

augmentation of simple model with $\alpha(V(s_j) - \dot{x}_j)$

$V(s_j)$ Optimal velocity function gives the speed each vehicle would want to go given the gap to lead vehicle

s_j small $V(s_j) \to 0$
s_j large $V(s_j) \to$ maximum allowable speed

Questions

1) What was your main takeaway message from the video?

2) In the follow leader model, what is the long time behavior of the vehicle's velocities?

3) What can you say about the spacing between the vehicles at equilibrium?

4) Follow the leader model: dynamically stable or unstable?

5) What does stable mean?

uniform flows an attractor of dynamics
start nearby remain nearby
start faraway approach system state
no equispaced cars

B) Optimal velocity encoded by V
10m/s

vehicles tend towards equispaced set up

roughly uniform speed (uniform flow)

uniform flow state — add pertubation
randomness in choice of initial velocity
1 and half to 2 minutes → see instabilities

6) What does unstable mean?

7) When vehicles aim at a particular optimal speed, the vehicles gravitate towards equispaced orientation?

When instabilities occur, some cars are moving at slower speeds than optimal "traffic jams"

✗ 8) What happens if instead of starting velocities at
5 + 10* rand(size(q))
we start at rand(size(q))

✗ 9) Plot V function from 0 to 20 ✓

✗ 10) Try Euler step with dt = 0.5
What happens & why?

traffic wave

traveling disturbances in distribution of cars on highway.

Wave speed distance wave travels in given amount of time

$$\text{speed} = \frac{\text{wavelength}}{\text{frequency}}$$

crest move distance of 20 m in 10 seconds