Populatio	<u>n Dynamic</u>	s Theory	 	• •	• • •	• •	• • •
Study	of why	& how	popul	atens	change	in sil	te L
nature	over tim	e	• • •	• •	• • •	• •	• • •
Simple	model	• • •	• • •	• •	• • •	• •	• • •
• • •	Doterministic) non-c	igent	Dased		• •	
Conside	r a si	ngle spe	icie i	denored	t by	x(t)	• • •
xl	t) —	population	, at	time	t.	• •	• • •
Nodi	l route of	f Change	of a	popu	lation	• •	••••
• • •	x (t) =	· · · ·	(+) –	S x (t)	• •	• • •
• • •	X'(+)	· · · ·	• • •	. (.	fl. c.ul a	• •	• • •
• • •	<u>dx</u>	birth	rate		the role.	• •	• • •
• • •	dt	• • •	• • •	• •	• • •	• •	• • •
	of channe	· · ·	ou Laber	 B	ue to	S ODAR	• • •
of	the popu	laten g	etting	born	and	some	dying
• • •	• • • •	• • •		• •	• • •	• •	
• • •	× (t) =	(B-8)×		• •	• • •	• •	• • •
••••	· · · · · ·	J X(t)	• • •	• •	••••	• •	• • •
• • •		's growt	h rate	• •	• • •	• •	• • •
• • •			TER	• •	• • •	• •	• • •
• • •		• • •	• • •	• •	• • •	• •	• • •

Has unique solution $\frac{dx}{dt} = \lambda x$ $\int \frac{1}{x} dx = \int \lambda dt$ $lnix = \lambda t + C$ $l^{lnix} = l^{\lambda t} + C$ $l^{lnix} = l^{\lambda t} + c^{\lambda}$ $k = l^{\lambda t} e^{c}$ $X = l^{\lambda t} e^{c}$ $K_{nuv} = \chi(0) = x_{0} = K$ $\chi(t) = x_{0} e^{\lambda t}$ See for different values of lambda Dynamics for $\lambda = 0$ $k = l^{\lambda t} e^{\lambda t}$	· · · · ·	× = x (0)	λx = X ₀	ODE initial	conditio	× .	• •	· ·
$\frac{dx}{dt} = \lambda x$ $\int \frac{1}{x} dx = \int \lambda dt$ $\ln x = \lambda t + C$ $e^{\ln x} = e^{\lambda t} + C$ $e^{\ln x} = e^{\lambda t} e^{C},$ $X = e^{\lambda t} e^{C},$ K $k(nw) = \chi_0 = K$ $x(t) = \chi_0 e^{\lambda t}$ $x(t) = \chi_0 e^{\lambda t}$ See for different values of lambda $Dynamics \text{ for } \chi \uparrow$ $exponential given the second secon$	Has	s unique	solution			• • •	• •	• •
dt $\int \frac{1}{X} dx = \int \lambda dt$ $lnx = \lambda t + C$ $e^{lnx} = e^{\lambda t} + c^{2}$ $X = e^{\lambda t} e^{c}$ K $k_{nw} \chi(0) = X_{0} = K$ $x(t) = X_{0} e^{\lambda t}$ See for different values of lambda Dynamics for $X \uparrow$ $exponential greater$		<u>dx</u>	= λ×				• •	• •
$\int \frac{1}{x} dx = \int \lambda dt$ $lnx = \lambda t + C$ $\ell^{lnx} = \ell^{\lambda t} + C$ $\chi = \ell^{\lambda t} e^{C}$ K $\ell(nu) = \chi_{0} = K$ K $\chi(t) = \chi_{0} e^{\lambda t}$ $\chi(t) = \chi_{0} e^{\lambda t}$ See for different values of lambda $\int y_{namics} fr$ $\chi \uparrow$ $expensential growth$		dt						
$lnx = \lambda t + C$ $e^{lnx} = e^{\lambda t} e^{C}$ $X = e^{\lambda t} e^{C}$ $Knw x(0) = x_{0} = K$ $x(t) = x_{0} e^{\lambda t}$ $See for different values of lambda$ $Dynamics for X f (exponential growth)$	• • • •	$\int \frac{1}{x}$	$dx = \int \lambda dt$	* * *	• • • •		• •	• •
$e^{\ln x} = e^{\lambda t} e^{c},$ $X = e^{\lambda t} e^{c},$ $K_{nu} = \chi_{0} = \chi_{0} = K$ $\chi(t) = \chi_{0} e^{\lambda t}$ See for different values of lambda $y_{nanvics, for} = \chi_{1}$ $exponential growth$	• • • •	lnx	=	+ C	• • • •	• • •	• •	• •
$x = e^{\lambda t} e^{C},$ $K_{\text{hwo}} \chi(0) = X_{0} = K$ $\chi(t) = X_{0} e^{\lambda t}$ $X(t) = X_{0} e^{\lambda t}$ See for different values of lambda $y_{\text{namics for}} \chi \uparrow$ $(exponential growth)$	· · · ·		x{\lambda t	+ C)		• • •	• •	• •
$X = e^{\pi c} e^{C}$ K			- 1)-	 بر			• •	• •
$K_{\text{nun}} \chi(0) = X_{0} = K$ $\chi(t) = \chi_{0} e^{\chi t}$ See for different values of lambda $V_{\text{nunvics for}} \chi \uparrow \qquad \text{exponential greated}$		X.	= e [~] e	K			• •	• •
$X(t) = X_0 e^{\chi t}$ See for different values of lambda Dynamics for $\chi \uparrow$ (exponential growth	12n	ω, χ(D) :=	= .X ₀ , =, K			• • •		• •
See for different values of lambda Dynamics for X / exponential growth	· · · · ·	χ(ŧ	$= x_0 e^7$	xt	· · · ·		• •	• •
2>0 X (exponential grewon	See	for di	fferent Vali	res of	lambda		• •	• •
	λ	>O X		/ expor	ientral g	ewth	• •	• •
$\chi = 0$	λ.	≈0 	· · · · ·		• • • •	• • •	• •	• •
2<0 Xo Static	j j j	<0 X0			Stutic	• • •	• •	• •
exponentral decay					exp	ioneintril	decar	7

In reality for 7>0 you don't get exponential grenster. There is limited resources space food etc. To model effects of overcrowding & limited resources biologist & demographers assume sufficiently get When X large. the $\frac{X}{x}$ decreases. growth rate capita per Xmax (carrying capacity Say X can grow only ኯ Growth Per λ C apida X Xmax greater than Xmax ulations growth rate fin negative linearly . <u>X</u> . X decreases with

$\dot{X} = -\frac{\lambda}{X_{max}} x^2 + \lambda x = \lambda x \left(1 - \frac{1}{X_{max}} \right)$ const μ	· · ·
	• •
$X(t) = \eta_X - \eta_X^2$ de logistic equicition	• •
The equation can be solved analytically but	••••
we can look at the vector field to see	• •
dynamics too. Plot x vs. x	• •
$\dot{\mathbf{x}}$	• •
	• •
	• •
	••••
$\frac{X_{\max}}{2}$	• •
$ \ldots \ldots$	• •
fixed points $\dot{X} = 0$	• •
$\chi = 0$ or $\chi = \chi_{max}$	• •
	• •



Τw	<u>o species</u>
1)	Lotka-Volterra predator-prey
• •	· · · · · · · · · · · · · · · · · · ·
• •	
• •	$\dot{X} = \alpha \cdot x - \beta x y = x(\alpha - \beta y)$
• •	$\dot{y} = \chi_{xy} - \delta_{y} = y(\chi_{x} - \delta)$
• •	birth death by
, , , ,	Obe system of equations Vonlinear
) O	$\alpha, \beta, \gamma, \beta > 0$
Un	realistic assumptions?
• •	I) prey has enough food at all threes
• •	2) food of predator depends on prey only
• •	3) rate of change of population proportional to size
• •	4) predutors have limitless appearte