Temple Math 5044 **Introduction to Numerical Analysis II** **Spring 2023**

Problem Set 6

(Out Tue 04/18/2023, Due Tue 04/25/2023)

Problem 10

Consider the Matlab program `temple5044_euler_maruyama.m` on the course website, which solves the geometric Brownian motion SDE

$$\mathrm{d}X = \mu X \,\mathrm{d}t + \sigma X \,\mathrm{d}W \ ,$$

that can be seen as a model for a stock price evolution.

(a) Add the Milstein method to the code and display the convergence rates in comparison to the Euler-Maruyama method.

Now consider the mean-reverting Ornstein-Uhlenbeck process

$$\mathrm{d}X = \mu X \,\mathrm{d}t + \sigma \,\mathrm{d}W \ ,$$

which is also the overdamped Langevin equation in a harmonic oscillator potential.

(b) Derive the true solution of this SDE (consult suitable sources to learn how to do so).

(c) Adapt your Matlab program from above to produce a numerical convergence study of the Euler-Maruyama method and the Milstein method. What is different to the geometric Brownian motion example?

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter.