

Problem Set 4

(Out Thu 03/16/2023, Due Thu 03/30/2023)

Problem 7

Consider the Prothero–Robinson test problem

$$\begin{cases} u'(t) = \lambda(u(t) - \phi(t)) + \phi'(t) \\ u(0) = \phi(0) \end{cases}$$

with $\phi(t) = \sin(t)$ and $\lambda = -10^4$ on $t \in [0, 1]$.

Write a Matlab program that produces the error convergence plot (in loglog scale) for $10^{-6} \leq k \leq 10^0$ for the various schemes given below. For each scheme, read off the non-stiff convergence order and the stiff-convergence order. Then, for each scheme, calculate the order and the stage order, and report whether the observed stiff-convergence order is in agreement with what order and stage order would indicate.

(a) Crank-Nicolson

$$\begin{array}{c|cc} 0 & 0 & \\ 1 & 1/2 & 1/2 \\ \hline & 1/2 & 1/2 \end{array}$$

(b) The TR-BDF2 method

$$\begin{array}{c|ccc} 0 & 0 & & \\ 1/2 & 1/4 & 1/4 & \\ 1 & 1/3 & 1/3 & 1/3 \\ \hline & 1/3 & 1/3 & 1/3 \end{array}$$

(c) The 5-stage stiffly accurate DIRK

$$\begin{array}{c|cccccc} 1/4 & 1/4 & & & & \\ 3/4 & 1/2 & 1/4 & & & \\ 11/20 & 17/50 & -1/25 & 1/4 & & \\ 1/2 & 371/1360 & -137/2720 & 15/544 & 1/4 & \\ 1 & 25/24 & -49/48 & 125/16 & -85/12 & 1/4 \\ \hline & 25/24 & -49/48 & 125/16 & -85/12 & 1/4 \end{array}$$

(d) The 4-stage stiffly accurate DIRK scheme

$$\begin{array}{c|cccccc} 0.13756543551 & 0.13756543551 & & & & \\ 0.80179011576 & 0.56695122794 & 0.23483888782 & & & \\ 2.33179673002 & -1.08354072813 & 2.96618223864 & 0.44915521951 & & \\ 1 & 0.59761291500 & -0.43420997584 & -0.05305815322 & 0.88965521406 & \\ \hline & 0.59761291500 & -0.43420997584 & -0.05305815322 & 0.88965521406 & \end{array}$$

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where 1 stands for the problem number and a for the sub-problem letter.