Temple 9200 **Computational Methods for Flow Problems** **Fall 2022**

Problem Set 4

(Out Thu 10/06/2022, Due Tue 10/18/2022)

Problem 4

Consider the Lighthill-Whitham-Richards (LWR) model for traffic flow $\rho_t + f(\rho)_x = 0$, where $f(\rho) = \rho(1-\rho)$ is the Greenshields flux, and initial conditions

$$\rho(x,0) = \begin{cases} 0.3 & x \le 0 \\ 0.6 & 0 < x \le 1 \\ 0.9 \quad \text{for} & 1 < x \le 2 \\ 0.6 & 2 < x \le 3 \\ 0.4 & 3 < x \, . \end{cases}$$

a) Use the software *particleclaw*, available on
`https://math.temple.edu/~seibold/research/particleclaw/` to solve the LWR problem given above. Email your program `yourfamilyname_problem4a.m` (that can be run together with the *particleclaw* solver file) that produces an animation of the solution on $x \in [-1, 8]$ for $t \in [0, 20]$.

b) Find the true solution of the problem at $t = 20$.[1]

c) Start with the resolution `parameters.d = [0 1e-1 2e-1]` . Then successively refine the resolution by powers of 2. For each computation, evaluate the L^1 error. Visualize the error convergence rate, by plotting (in log-log scale) the L^1 error vs. the third argument of `parameters.d = [0 1e-1 2e-1]` . Email your program `yourfamilyname_problem4c.m` . What convergence order do you observe, and how does it compare to the convergence order obtained with fixed-grid finite volume methods?[1]

Problem 5

Download the Matlab files `particleclaw_network_ex_diamond.m` and `particleclaw_network_solver.m` from the course website, and run the former file. Then, write a program that solves the same problem, but by using a simple Godunov method on each edge, and the standard vertex coupling conditions discussed in class. You can use the plotting routines from the `particleclaw` files for the output of the numerical solutions of your programs.

Problem P1

The task of this practice problem is for the assigned team of students to get the *Clawpack* software, hosted on `http://www.clawpack.org` , to run, to prepare several representative and interesting examples of hyperbolic conservations laws and systems in various dimensions, and to give a short talk about the experience to the rest of the class.

[1] You are allowed to use solutions from past courses here.