Temple 8024 **Numerical Differential Equations II** **Spring 2022**

Problem Set 4

(Out Thu 03/10/2022, Due Tue 03/22/2022)

Problem 5

Download the Matlab file `temple8024_godunov_shallow_water.m` from the course website.

a) Run the code, and explain what you see (which problem is solved, what scheme is used, etc.).

b) Modify the code to solve *Moses' first problem*:

$$h(x,0) = 1 \qquad \text{and} \qquad u(x,0) = \begin{cases} -0.88 & \text{for } x < 0 \\ 0.88 & \text{for } x \geq 0 \end{cases}$$

on the domain $x \in [-5, 5]$. Plot the height and velocity field at $t = 2$. Explain what physically should happen, how the numerical solution looks like, any where any discrepancies stem from.

c) Now change the code to solve *Moses' second problem*:

$$h(x,0) = \begin{cases} 1 & \text{for } x \leq -2 \\ 0.01 & \text{for } -2 < x < 2 \\ 1 & \text{for } x \geq 2 \end{cases} \qquad \text{and} \qquad u(x,0) = 0$$

on the domain $x \in [-5, 5]$. Plot the height and velocity field at $t = 2.5$. Again, explain the physical behavior of the true solution, the shape of the numerical solution, and the reason for any discrepancies.

d) [Bonus problem] Explain in which sense issues with entropy arise in the examples above. Add an entropy fix to the Matlab code, re-run the three examples above, and demonstrate the benefit of your fix.

Problem 6

(1) Download the code `temple8024_weno_claw.m` from the course website and run it. Plot the numerical approximation obtained with 80 cells, together with the true solution.

(2) Change the code so that it approximates the linear advection equation with smooth initial conditions. Perform a numerical error analysis and report the scheme's convergence rate. Explain why we do not obtain fifth order, even though a fifth order WENO reconstruction is used.