Temple 8024 **Numerical Differential Equations II** **Spring 2022**

Problem Set 2

(Out Thu 02/03/2022, Due Tue 02/15/2022)

Problem 3

Consider the Lighthill-Whitham-Richards (LWR) model for traffic flow $\rho_t + f(\rho)_x = 0$, where $f(\rho) = \rho(1-\rho)$ is the Greenshields flux, and initial conditions

$$\rho(x,0) = \begin{cases} 0.3 & x \le 0 \\ 0.6 & 0 < x \le 1 \\ 0.9 \quad \text{for} & 1 < x \le 2 \\ 0.6 & 2 < x \le 3 \\ 0.4 & 3 < x \, . \end{cases}$$

a) Calculate/construct analytically the true solution of the problem at $t = 20$.

b) Use the software *particleclaw*, available on
`https://math.temple.edu/~seibold/research/particleclaw/` to solve the LWR problem given above. Use the program to plot the numerical solution on $x \in [-1, 8]$ for $t \in \{1, 2, 5, 10, 20\}$.

c) Using *particleclaw*, conduct a numerical convergence analysis. To do so, start with the resolution `parameters.d = [0 1e-1 2e-1]` . Then successively refine the resolution by powers of 2. For each computation, evaluate the L^1 error on $x \in [-1, 8]$ for $t = 20$ (using the true solution from part a)). Using those errors, visualize the convergence rate of the method, by plotting (in log-log scale) the L^1 error vs. the third argument of `parameters.d = [0 1e-1 2e-1]` .

d) Now implement Godunov's method for the problem. Using a cell size of $h = 0.1$, run the method (with time step $k = 0.1$) on $x \in [-1, 8]$ until $t = 20$. Plot the numerical approximation at $t \in \{1, 2, 5, 10, 20\}$.

e) Analogous to part c), conduct a numerical convergence analysis of the Godunov method. Explain your observations.