Temple 8023 **Numerical Differential Equations I** **Fall 2021**

Problem Set 4

(Out Thu 10/14/2021, Due Thu 10/28/2021)

Problem 8

We would like to investigate whether it is better to install a radiator under a window or at the opposite wall. Consider the evolution of temperature $u(x, y, t)$ be described by the heat equation

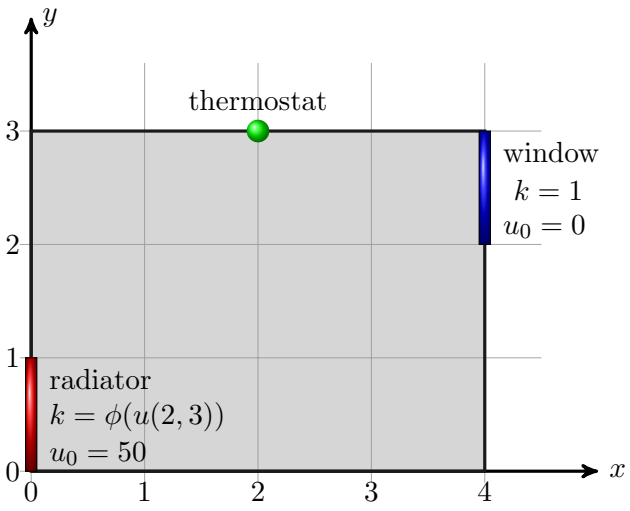
$$\begin{cases} u_t = \nabla^2 u & \text{in } \Omega \\ \frac{\partial u}{\partial n} = k(u_0 - u) & \text{on } \partial\Omega \\ u(x, y, 0) = 0 \end{cases}$$

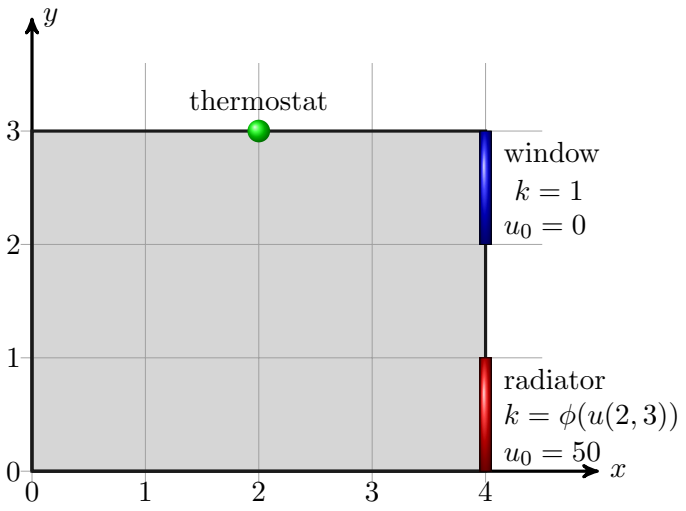
in a 2D room $\Omega =]0, 4[\times]0, 3[$, where

- $u_0 = 0$ and $k = 1$ at a window $\{4\} \times [2, 3]$,
- $u_0 = 50$ and $k = \phi(u(2, 3))$ at the radiator $\{0\} \times [0, 1]$ or $\{4\} \times [0, 1]$, and
- $k = 0$ (perfect insulation) everywhere else at the boundary.

The radiator's heating properties depend on the temperature at a thermostat via

$$\phi(u) = \begin{cases} 10 & u \le 20 \\ 0 & u > 20 \, . \end{cases}$$





(1) Write a program that approximates the true solution with sufficient accuracy in space and time, and simulate both rooms.

(2) In both rooms, plot the time evolution of the temperature at $P = (1, 2)$ and $Q = (3, 2)$. Choose the final time large enough that you see a time-periodic behavior towards the end.

(3) What is the average temperature in each room (averaged over space and over one periodic cycle)?

(4) Which room is more comfortable to be in, and why?

(5) Which design is more energy-efficient (the loss of energy is the integral over $\frac{\partial u}{\partial n}$ over the window)?