Temple 8023 **Numerical Differential Equations I** **Fall 2021**

Problem Set 2

(Out Thu 09/23/2021, Due Thu 10/07/2021)

Problem 6

Consider the steady-state convection-diffusion equation

$$-\varepsilon u_{xx} + u_x = 1$$

in $]-1,1[$ with $u(-1) = 0 = u(1)$. Write a program that uses not more than 500 grid points and that, for each choice of $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$, approximates the solution with acceptable accuracy. Visualize your computational grids, and plot your numerical approximation, and its difference to the true solution.

Problem 7

On the unit square $\Omega =]0,1[\times]0,1[$, consider the 2D anisotropic steady diffusion problem

$$\begin{cases} 100u_{xx} + u_{yy} = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

Write a program that approximates the solution of this problem, and test it on the following examples:

1. For $f(x,y) = \begin{cases} 1 & \text{if } x < \frac{1}{2}, y > \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$, plot the numerical solution with 100 grid points in each dimension.
2. Calculate the forcing $f(x,y)$ that generates the solution $u(x,y) = \sin(3\pi x)\sin(\pi y)$. Then, apply your code on this problem with $h \in \{1/50, 1/100, 1/200, 1/400\}$, calculate the approximation error, and plot it as a function of h in log–log scale.[1] What is the convergence order of your method?

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[1] This methodology (pick u and then calculate the corresponding forcing f) is called "method of manufactured solutions", and it is a great way to generate analytical solutions to complicated problems, as needed, for example, for numerical convergence studies.