Temple 8024 **Numerical Differential Equations II** **Spring 2020**

Problem Set 5

(Out Wed 03/18/2020, Due Mon 03/30/2020)

Problem 6

(1) Download the code `temple8024_weno_claw.m` from the course website and run it. Plot the numerical approximation obtained with 80 cells, together with the true solution.

(2) Change the code so that it approximates the linear advection equation with smooth initial conditions. Perform a numerical error analysis and report the scheme's convergence rate. Explain why we do not obtain fifth order, even though a fifth order WENO reconstruction is used.

Problem 7

Write an at least third order accurate WENO code[1] that solves the 2D advection equation

$$\phi_t + u\phi_x + v\phi_y = 0 \,, \quad (x, y) \in]0, 1[^2, t \in]0, T[$$

with the velocity field $(u, v) = (-\psi_y, \psi_x)$ where $\psi(x, y, t) = \cos(\pi t/T) \sin^2(\pi x) \sin^2(\pi y)$, with $T = 4$.

Run your code on the initial conditions $\phi(x, y, 0) = \sqrt{(x - 0.25)^2 + (y - 0.3)^2} - 0.1$, and plot the zero contour $\Gamma(t) = \{(x, y) : \phi(x, y, t) = 0\}$ at times $t \in \{0, 1, 2, 3, 4\}$. Do so for three resolutions: one for which the results look bad, one for which they look descent, and one for which they look very good.

[1] Use the code `temple8024_weno_claw.m` for inspiration.