Temple 8024 **Numerical Differential Equations II** **Spring 2020**

Problem Set 3

(Out Mon 02/17/2020, Due Mon 02/24/2020)

Problem 4

Download the Matlab file `mit18086_fd_transport_limiter.m` from the course website. Apply the program (with $k = 0.9\,h$) to the linear advection equation

$$\begin{cases} u_t + u_x = 0 \\ u(x,0) = u_0(x) \end{cases}$$

on $x \in [0,1]$ with periodic boundary conditions, and $t \in [0,1]$, with the following sets of initial conditions:

(A) $u(x,0) = \sin(2\pi x)$

(B) $u(x,0) = \begin{cases} x & x \in [0, \frac{1}{2}) \\ 1-x & x \in [\frac{1}{2}, 1) \end{cases}$

(C) $u(x,0) = \begin{cases} 1 & x \in [0, \frac{1}{2}) \\ 0 & x \in [\frac{1}{2}, 1) \end{cases}$

a) As in Problem 1 (problem set 1), conduct a numerical error analysis (in $\|\cdot\|_\infty$ and $\|\cdot\|_1$) of the method with the following schemes/limiters: (i) none (upwind); (ii) none (Lax-Wendroff), (iii) superbee, (iv) van-Leer, (v) minmod [you need to add minmod to the program]. Explain your observations.

b) Now change the final time to $t = 10$, and re-do the convergence study. Explain your observations.