

Problem Set 2

(Out Mon 02/03/2020, Due Wed 02/12/2020)

Problem 3

Consider the Lighthill-Whitham-Richards (LWR) model for traffic flow $\rho_t + f(\rho)_x = 0$, where $f(\rho) = \rho(1 - \rho)$ is the Greenshields flux, and initial conditions

$$\rho(x, 0) = \begin{cases} 0.3 & x \leq 0 \\ 0.6 & 0 < x \leq 1 \\ 0.9 & \text{for } 1 < x \leq 2 \\ 0.6 & 2 < x \leq 3 \\ 0.4 & 3 < x \end{cases}$$

- a) Calculate/construct analytically the true solution of the problem at $t = 20$.
- b) Use the software *particleclaw*, available on <https://math.temple.edu/~seibold/research/particleclaw/> to solve the LWR problem given above. Use the program to plot the numerical solution on $x \in [-1, 8]$ for $t \in \{1, 2, 5, 10, 20\}$.
- c) Using *particleclaw*, conduct a numerical convergence analysis. To do so, start with the resolution `parameters.d = [0 1e-1 2e-1]`. Then successively refine the resolution by powers of 2. For each computation, evaluate the L^1 error on $x \in [-1, 8]$ for $t = 20$ (using the true solution from part a)). Using those errors, visualize the convergence rate of the method, by plotting (in log-log scale) the L^1 error vs. the third argument of `parameters.d = [0 1e-1 2e-1]`.
- d) Now implement Godunov's method for the problem. Using a cell size of $h = 0.1$, run the method (with time step $k = 0.1$) on $x \in [-1, 8]$ until $t = 20$. Plot the numerical approximation at $t \in \{1, 2, 5, 10, 20\}$.
- e) Analogous to part c), conduct a numerical convergence analysis of the Godunov method. Explain your observations.