Temple Math 5044 **Introduction to Numerical Analysis II** **Spring 2019**

Problem Set 7

(Out Wed 03/25/2019, Due Wed 04/03/2019)

Problem 7

Consider the Prothero–Robinson test problem

$$\begin{cases} u'(t) &= \lambda(u(t) - \phi(t)) + \phi'(t) \\ u(0) &= \phi(0) \end{cases}$$

with $\phi(t) = \sin(t)$ and $\lambda = -10^4$ on $t \in [0, 1]$.

Write a Matlab program that produces the error convergence plot (in loglog scale) for $10^{-6} \leq k \leq 10^0$ for the various schemes given below. For each scheme, read off the non-stiff convergence order and the stiff-convergence order. Then, for each scheme, calculate the order and the stage order, and report whether the observed stiff-convergence order is in agreement with what order and stage order would indicate.

(a) Crank-Nicolson

0	0	
1	1/2	1/2
	1/2	1/2

(b) The TR-BDF2 method

0	0		
1/2	1/4	1/4	
1	1/3	1/3	1/3
	1/3	1/3	1/3

(c) The 5-stage stiffly accurate DIRK

1/4	1/4				
3/4	1/2	1/4			
11/20	17/50	−1/25	1/4		
1/2	371/1360	−137/2720	15/544	1/4	
1	25/24	−49/48	125/16	−85/12	1/4
	25/24	−49/48	125/16	−85/12	1/4

(d) The 4-stage stiffly accurate DIRK scheme

0.13756543551	0.13756543551			
0.80179011576	0.56695122794	0.23483888782		
2.33179673002	−1.08354072813	2.96618223864	0.44915521951	
1	0.59761291500	−0.43420997584	−0.05305815322	0.88965521406
	0.59761291500	−0.43420997584	−0.05305815322	0.88965521406

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter.