Problem Set 5

(Out Wed 02/20/2019, Due Wed 03/13/2019)

Problem 5

Consider the linear ODE system

$$\begin{cases} \vec{u}'(t) = A \cdot \vec{u}(t) \\ \vec{u}(0) = \mathring{\vec{u}} \end{cases} \tag{1}$$

where

$$A = \begin{pmatrix} -5000 & 4999 & 0 & 0 \\ 4999 & -5000 & 0 & 0 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 10 & 0 \end{pmatrix} \quad \text{and} \quad \mathring{\vec{u}} = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

We would like to approximate the solution of (1) on $t \in [0, 1]$, using an ODE solver with equidistant time steps. We are happy to be within 5% accuracy.

(1) Calculate the true solution $\vec{u}(t)$.

(2) Implement the following time-stepping schemes and apply them to this problem:

   (a) forward Euler
   (b) backward Euler
   (c) RK4
   (d) Crank-Nicolson
   (e) Adams-Bashforth 4 [1]
   (f) Adams-Moulton 4 [1]
   (g) BDF2
   (h) BDF4 [1]

   Then determine, for each scheme, numerically the maximum time step that yields the desired accuracy.

(3) Explain your observations.

(4) For the largest time step that yields the desired accuracy for backward Euler, $k_{\text{BE}}^{\max}$ plot the numerical solutions obtained with each scheme, when using that same time step $k_{\text{BE}}^{\max}$ (plot also the true solution in the same figure, and limit the u-axis to $[-2, 2]$).

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter.

[1] Cheat and use the correct solution for the first $k - 1$ steps.