Temple Math 5044 **Introduction to Numerical Analysis II** **Spring 2019**

Problem Set 2

(Out Mon 01/28/2019, Due Mon 02/04/2019)

Problem 2

Consider the Lotka-Volterra predator-prey model from Problem 1:

$$\frac{d}{dt}\vec{u}(t) = \vec{f}(\vec{u}(t)) \;, \tag{1}$$

where $\vec{u} = \begin{pmatrix} u \\ v \end{pmatrix}$ and $\vec{f}(\vec{u}) = \begin{pmatrix} f_1(u,v) \\ f_2(u,v) \end{pmatrix} = \begin{pmatrix} u - 4uv \\ -v + 5uv \end{pmatrix}$.

(a) Calculate the Jacobian matrix $D\vec{f}(\vec{u}) = \begin{pmatrix} \frac{\partial f_1}{\partial u}(u,v) & \frac{\partial f_1}{\partial v}(u,v) \\ \frac{\partial f_2}{\partial u}(u,v) & \frac{\partial f_2}{\partial v}(u,v) \end{pmatrix}$, and the product $D\vec{f}(\vec{u}) \cdot \vec{f}(\vec{u})$.

(b) Use the expressions from (a) to formulate a second order Taylor series method for (1).

(c) Start with the results produced in parts (d) and (e) in Problem 1 (previous problem set), and add the analogous plots produced with the Taylor series method formulated above (using the same parameters, but using different colors, line styles, and/or labels for the two methods). Explain your observations.

(d) Use the Taylor series method to approximate the time T that it takes for a trajectory $\vec{u}(t)$ to return (for the first time) to its initial value, i.e. $\vec{u}(T) = \vec{u}(0)$. Do this for at least 50 initial values

$$\vec{u}(0) = \begin{pmatrix} a \\ 0.25 \end{pmatrix} \text{ with } 0 < a < 0.2$$

and thus produce a plot of the function $T(a)$.

(e) Explain the fact that for $a \approx 0.25$, it takes close to $T = 2\pi$ for the solution to "go around" once.

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter.