Temple Math 5044 **Introduction to Numerical Analysis II** **Spring 2019**

Problem Set 1

(Out Wed 01/16/2019, Due Mon 01/28/2019)

Problem 1

Consider a Lotka-Volterra predator-prey model for a population of carps and pikes, whose numbers are given by $u(t)$ and $v(t)$, respectively. The growth/decay rates are given by

$$\begin{aligned} \frac{du}{dt} &= \quad u - 4uv \\ \frac{dv}{dt} &= -v + 5uv \end{aligned} \tag{1}$$

(a) Show that the function $H(u, v) = uv \exp(-5u-4v)$ is a *constant of motion*, i.e. if $(u(t), v(t))$ is a solution of (1), then $H(u(t), v(t))$ is constant in time.

(b) Using Matlab's `mesh` (or similar) command, plot the function H on the domain $(u, v) \in [0, 1]^2$.

(c) Using Matlab's `quiver` command, plot the velocity field given by the right hand side vector of (1), scaled to length 1 everywhere. On top of this plot, overlay isocontours of the function H, using Matlab's `contour` command.

(d) Starting with $u(0) = 0.2$ and $v(0) = 0.8$, approximate (1) using Euler's method for $t \in [0, 8]$. Use steps of size $\Delta t = 0.01$. Plot all 801 points obtained from this numerical solution into the figure created in (c). Explain why the resulting curve is not closed.

(e) Run the same computation with $\Delta t = 0.02$ and $\Delta t = 0.04$, as well as $\Delta t = 0.005$ and $\Delta t = 0.0025$. How does the observed approximation error behave. Explain your observations.

Instructions

For each problem set, you need to submit one document, either in class or via email to the course instructor, that contains plots and explanations (hand-written or typed). If you decide to email the document, name it `yourfamilyname_problemset1.pdf`, where 1 stands for the number of the problem set.

In addition, for each programming task, email your respective program to the course instructor, under the filename `yourfamilyname_problem1a.m`, where `1` stands for the problem number and `a` for the sub-problem letter. [On this problem set, you need to submit four Matlab codes, for `1b`, `1c`, `1d`, and `1e`.]