**Temple 8024** **Numerical Differential Equations II** **Spring 2017**

**Problem Set 5**

(Out Thu 03/30/2017, Due Tue 04/18/2017)

## Problem 9

Consider the steady-state convection-diffusion equation

$$-\varepsilon u_{xx} + u_x = 1$$

in $]-1,1[$ with $u(-1) = 0 = u(1)$.

(1) Write a spectral code based on Chebyshev points that approximates the solution. Test your code for $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$, and show for each value of $\varepsilon$ the error convergence. Give sufficient attention to small numbers of grid points, and explain the observed error behavior as the resolution increases.

(2) Design a scheme for this problem that is spectrally accurate, and works with a small number of grid points even for $\varepsilon \leq 10^{-5}$.

## Problem 10

Write a finite difference method for the Stokes problem

$$\begin{cases} -\nabla^2 \vec{u} + \nabla p = \vec{f} & \text{in } \Omega \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega \\ \vec{u} = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega = ]0,1[^2$. Use your code to compute the velocity fields for the following force fields

(1) $\vec{f}(\vec{x}) = (\vec{x} - \vec{x}_0)\exp(-50\|\vec{x} - \vec{x}_0\|^2)$, where $\vec{x}_0 = (0.4, 0.3)$.

(2) $\vec{f}(\vec{x}) = (0, x_1(1 - x_1))$

(3) $\vec{f}(\vec{x}) = (x_1(1 - x_1), 0)$ (Here, be careful to interpret the results correctly.)

Feel free to use the code `mit18086_navierstokes.m` as a starting point.